Differentiation

Formulae:

- $\frac{d}{dx}(\text{constant}) = 0$
- $\frac{d}{dx}(x^n) = n \cdot x^{n-1}$
- $\frac{d}{dx}(A \cdot x^n) = A \cdot n \cdot x^{n-1}$
- $\frac{d}{dx}(x) = 1$
- $\frac{d}{dx}(e^x) = e^x$
- $\frac{d}{dx}(a^x) = a^x \cdot \log a$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
- $\frac{d}{dx}(\sec x) = \sec x . \tan x$
- $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x . \cot x$
- $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
- $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$
- $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$
- $\frac{d}{dx}(\operatorname{cosec}^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$

$\rightarrow \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

$\rightarrow \frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$

$\rightarrow \frac{d}{dx}(\log_a x) = \frac{1}{x . \log a}$

$\rightarrow \frac{d}{dx} \cdot \frac{1}{x^n} = \frac{-n}{x^{n-1}}$

$\rightarrow \frac{d}{dx}(\log_e |x|) = \frac{1}{x}$

$\rightarrow \frac{d}{dx}(|x|) = \frac{|x|}{x}$

$\rightarrow \frac{d}{dx}(\sin hx) = \cos hx$

$\rightarrow \frac{d}{dx}(\cos hx) = \sin hx$

$\rightarrow \frac{d}{dx}(\tan hx) = \sec h^2 x$

$\rightarrow \frac{d}{dx}(\cot hx) = -\operatorname{cosec} h^2 x$

$\rightarrow \frac{d}{dx}(\sec hx) = -\sec hx . \tan hx$

$\rightarrow \frac{d}{dx}(\operatorname{cosec} hx) = -\operatorname{cosec} hx . \cot hx$

$\rightarrow \frac{d}{dx}(\sin h^{-1}x) = \frac{1}{\sqrt{1+x^2}}$

$\rightarrow \frac{d}{dx}(\cos h^{-1}x) = \frac{1}{\sqrt{x^2-1}}$

$\rightarrow \frac{d}{dx}(\tan h^{-1}x) = \frac{1}{1-x^2}$

$\rightarrow \frac{d}{dx}(\cot h^{-1}x) = \frac{1}{1-x^2}$

$\rightarrow \frac{d}{dx}(\sec h^{-1}x) = \frac{-1}{|x|\sqrt{1-x^2}}$

$\rightarrow \frac{d}{dx}(\operatorname{cosec} h^{-1}x) = \frac{-1}{|x|\sqrt{1+x^2}}$

Integrations

Formulae:

→ $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$

→ $\int x \, dx = \frac{x^2}{2} + c$

→ $\int (1) \, dx = x + c$

→ $\int (ax+b)^n \cdot dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$

→ $\int \frac{1}{x} \, dx = \log^{x} + c$

→ $\int \frac{1}{ax+b} \, dx = \frac{\log_e (ax+b)}{a} + c$

→ $\int e^x \, dx = e^x + c$

→ $\int e^{ax+b} \, dx = \frac{e^{ax+b}}{a} + c$

→ $\int a^x \cdot dx = \frac{a^x}{\log a} + c$

→ $\int k^{ax+b} \, dx = \frac{k^{ax+b}}{a \cdot \log k} + c$

→ $\int \log x \, dx = x \log x - x.$

→ $\int \sin x \, dx = -\cos x + c$

→ $\int \cos x \cdot dx = +\sin x + c$

→ $\int \sin(ax+b) \cdot dx = \frac{-\cos(ax+b)}{a} + c$

→ $\int \tan x \, dx = \log |\sec x| + c$

→ $\int \tan x \cdot dx = -\log |\cos x| + c$

$\rightarrow \int \tan(ax+b)\,dx = \dfrac{\log \sec(ax+b)}{a} + C$

$\rightarrow \int \cot x\,dx = \log|\sin x| + C$

$\rightarrow \int \sec x.dx = \log|\sec x + \tan x| + C$

$\rightarrow \int \operatorname{cosec} x\,dx = \log(\operatorname{cosec} x - \cot x) + C$

$\rightarrow \int (f(x))^n = \dfrac{f(x)^{n+1}}{n+1} + C$

$\rightarrow \int \dfrac{f'(x)}{f(x)}\,dx = \log|f(x)| + C$

$\rightarrow \int \dfrac{f'(x)}{\sqrt{f(x)}}\,dx = 2\sqrt{f(x)} + C$

$\rightarrow \int \dfrac{1}{x^2+a^2}\,dx = \dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right) + C$

$\rightarrow \int \dfrac{1}{1+x^2}\,dx = \tan^{-1}x + C$

$\rightarrow \int \dfrac{1}{a^2-x^2}\,dx = \dfrac{1}{2a}\log\left|\dfrac{a+x}{a-x}\right| + C$

$\rightarrow \int \dfrac{1}{x^2-a^2}\,dx = \dfrac{1}{2a}\log\left|\dfrac{x-a}{x+a}\right| + C$

$\rightarrow \int \dfrac{1}{\sqrt{x^2+a^2}}\,dx = \sinh^{-1}\left(\dfrac{x}{a}\right) + C$

(or)

$\log(x + \sqrt{x^2+a^2}) + C$

$\rightarrow \int \dfrac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\dfrac{x}{a}\right) + C$ (or) $-\cos^{-1}\left(\dfrac{x}{a}\right) + C$

$\rightarrow \int \dfrac{1}{\sqrt{x^2-a^2}}\,dx = \cosh^{-1}\left(\dfrac{x}{a}\right) + C$

(or)

$\log(x + \sqrt{x^2-a^2}) + C$

$$\rightarrow \int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$$

$$\rightarrow \int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right) + c$$

$$\rightarrow \int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\cosh^{-1}\left(\frac{x}{a}\right) + c$$

ILATE

$$\rightarrow \int e^{ax}.\sin bx.\,dx = \frac{e^{ax}}{a^2+b^2}\left[a.\sin bx - b.\cos bx\right] + c$$

$$\rightarrow \int e^{ax}\cos bx.\,dx = \frac{e^{ax}}{a^2+b^2}\left[a.\cos bx + b\sin bx\right] + c$$

U,V Formulae :

$$\rightarrow d(u \pm v) = d(u) \pm d(v)$$

$$\rightarrow d(u.v) = \cancel{d\,u.dv}\ u.dv + v.du.$$

$$\rightarrow d\left(\frac{u}{v}\right) = \frac{v.du - u.dv}{v^2}$$

Solutions of First Order Differential Equations And Applications.

(4) $\cos^2 x \dfrac{dy}{dx} + y = \tan x.$

Sol:- $\dfrac{\cos^2 x}{\cos^2 x}\dfrac{dy}{dx} + \dfrac{y}{\cos^2 x} = \dfrac{\tan x}{\cos^2 x}$

$$\frac{dy}{dx} + \frac{1}{\cos^2 x}\cdot y = \tan x\cdot\sec^2 x$$

$$\frac{dy}{dx} + \sec^2 x\cdot y = \tan x\cdot\sec^2 x. \rightarrow ①$$

Here $P = \sec^2 x$ $\quad Q = \tan x\cdot\sec^2 x$

Now, I.F $e^{\int P(x)dx} = e^{\int \sec^2 x\cdot dx}$

$= e^{\tan x}$

Now the solution of equ ① is

$$y\cdot e^{\tan x} = \int \tan x\cdot\sec^2 x\cdot e^{\tan x}dx + c$$

Let $\tan x = t$

$\sec^2 x\cdot dx = dt$

$$y\cdot e^{\tan x} = \int t\cdot e^t\cdot dt + c$$

$= t\cdot e^t - e^t + c$

$= e^t(t-1) + c$

D.	I.
$+t$	e^t
-1	e^t
0	e^t

$$\boxed{y\cdot e^{\tan x} = e^{\tan x}(\tan x - 1) + c.}$$

(2) $\left(\dfrac{e^{-2\sqrt{x}}}{\sqrt{x}} - \dfrac{y}{\sqrt{x}}\right)\dfrac{dx}{dy} = 1$

Sol:- $\dfrac{dx}{dy} = \dfrac{1}{\dfrac{e^{-2\sqrt{x}}}{\sqrt{x}} - \dfrac{y}{\sqrt{x}}}$

$$\frac{dy}{dx} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}.$$

$$\frac{dy}{dx} + \frac{y}{\sqrt{x}} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$$

$$\frac{dy}{dx} + \frac{1}{\sqrt{x}} \cdot y = \frac{e^{-2\sqrt{x}}}{\sqrt{x}} \rightarrow ①$$

where $P = \frac{1}{\sqrt{x}}$, and $Q = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$

Now, $I.F = e^{\int P(x)dx}$

$$= e^{\int \frac{1}{\sqrt{x}} dx}$$

$$= e^{\int x^{-1/2} dx}$$

$$= e^{\frac{x^{1/2}}{1/2}}$$

$$= e^{2\sqrt{x}}$$

Now the solution of equ ① is

$$y \cdot e^{2\sqrt{x}} = \int \frac{\cancel{e^{-2\sqrt{x}}}}{\sqrt{x}} \cdot \cancel{e^{2\sqrt{x}}} \, dx + c$$

$$= \int \frac{1}{\sqrt{x}} dx + c$$

$$= \int x^{-1/2} dx + c$$

$$= \frac{x^{1/2}}{1/2} + c$$

$$\boxed{y \cdot e^{2\sqrt{x}} = 2\sqrt{x} + c}$$

(13) $\frac{dy}{dx} = \frac{y}{2y \log y + y - x}$

Sol:- $\frac{dx}{dy} = \frac{2y \log y + y - x}{y}$

$$\frac{dx}{dy} = \frac{2\cancel{y} \log y}{\cancel{y}} + \frac{y}{y} - \frac{x}{y}$$

$$= 2 \log y + 1 - \frac{x}{y}$$

$$\frac{dx}{dy} + \frac{1}{y} \cdot x = 2 \log y + 1 \longrightarrow ①$$

where $p = \frac{1}{y}$., $Q = 2\log y + 1$

Now I.F $e^{\int P(y)\,dy}$

$= e^{\int \frac{1}{y}\,dy}$

$= e^{\log_e y} = y$

Now the solution of equ ① is

$x \cdot e^{\log y} = \int (2\log y + 1)\, e^{\log y}\, dy + c$

$x \cdot y = \int (2\log y + 1)\, y \cdot dy + c$

$xy = \int (2y \cdot \log y + y)\, dy + c$

$xy = 2\int y \cdot \log y \cdot dy + \int y \cdot dy + c$

$= 2\left[\log y \cdot \frac{y^2}{2} - \int \frac{1}{y} \cdot \frac{y^2}{2}\, dy\right] + \frac{y^2}{2} + c$

$= 2\left[\log y \cdot \frac{y^2}{2} - \frac{1}{2}\int y\, dy\right] + \frac{y^2}{2} + c$

$= 2\left[\log y \cdot \frac{y^2}{2} - \frac{1}{2} \cdot \frac{y^2}{2}\right] + \frac{y^2}{2} + c$

$= \log y \cdot y^2 - \frac{y^2}{2} + \frac{y^2}{2} + c$

$x \cdot y = y^2 \log y + c$

H.W

(3) $\frac{dy}{dx} + \frac{y}{x} = x^3 - 3$.

sol:-

$\frac{dy}{dx} + \frac{1}{x} \cdot y = x^3 - 3 \longrightarrow ①$

I.F $e^{\int P(x)\,dx} = e^{\int \frac{1}{x}\,dx}$

$= e^{\log_e x}$

$= x$.

Now, the solution of equ ① is

$y \cdot x = \int (x^3 - 3)\, x \cdot dx + c$

$= \int (x^4 - 3x)\, dx + c$

$xy = \frac{x^5}{5} - 3\frac{x^2}{2} + c$

(5) $x.\log x \frac{dy}{dx} + y = 2.\log x$

Sol:-

$$\frac{x.\log x}{x.\log x}.\frac{dy}{dx} + \frac{y}{x.\log x} = \frac{2\log x}{x.\log x}$$

$$\frac{dy}{dx} + \frac{1}{x.\log x}.y = \frac{2}{x} \rightarrow ①$$

where $P = \frac{1}{x.\log x}$, $Q = \frac{2}{x}$

I.F $e^{\int P(x)\,dx} = e^{\int \frac{1}{x.\log x} dx}$ put $\log x = t$

$\frac{1}{x}.dx = dt$

$$= e^{\int \frac{1}{t} dt}$$

$$= e^{\log_e t}$$

$$= t$$

$$= \log x$$

Now the solution of equ ① is

$$y.\log x = \int \frac{2}{x}.\log x.\,dx + c$$

$$= 2\int \frac{1}{x}.\log x.\,dx + c$$ put $\log x = t$

$\frac{1}{x}.dx = dt$

$$= 2\int t.dt + c$$

$$= \cancel{2}.\frac{t^2}{\cancel{2}} + c$$

$$= t^2 + c$$

$$\boxed{y.\log x = (\log x)^2 + c}$$

(6) $(1+x^3)\frac{dy}{dx} + 6x^2y = 1+x^2$

Sol:-

$$\frac{\cancel{(1+x^3)}}{\cancel{1+x^3}}\frac{dy}{dx} + \frac{6x^2y}{1+x^3} = \frac{1+x^2}{1+x^3}$$

$$\frac{dy}{dx} + \frac{6x^2}{1+x^3}.y = \frac{1+x^2}{1+x^3} \rightarrow ①$$

where $P = \frac{6x^2}{1+x^3}$, $Q = \frac{1+x^2}{1+x^3}$

I.F $e^{\int P(x)\,dx} = e^{\int \frac{6x^2}{1+x^3} dx}$

$$= e^{2\int \frac{3x^2}{1+x^3} dx}$$

$$= e^{2.\log(1+x^3)}$$

$$= e^{\log_e (1+x^3)^2}$$

$$= (1+x^3)^2$$

Now the solution of equ ① is

$$y\cdot(1+x^3)^2 = \int \frac{1+x^2}{(1+x^3)} (1+x^3)^2 dx + c$$

$$= \int (1+x^3+x^2+x^5)\, dx + c$$

$$y\cdot(1+x^3)^2 = x + \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^6}{6} + c$$

(7) $\frac{dy}{dx} + y\cdot \cot x = \cos x$

Sol:- $\frac{dy}{dx} + \cot x \cdot y = \cos x \longrightarrow ①$

where $p = \cot x$, $Q = \cos x$

I.F $e^{\int p(x)\, dx} = e^{\int \cot x\, dx}$

$$= e^{\log_e |\sin x|}$$

$$= \sin x$$

Now, the solution of equ ① is

$$y\cdot \sin x = \int \cos x \cdot \sin x \cdot dx + c$$

put $\sin x = t$

$\cos x \cdot dx = dt$

$$= \int t \cdot dt + c$$

$$= \frac{t^2}{2} + c$$

$$y\cdot \sin x = \frac{(\sin x)^2}{2} + c$$

(8) $(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$

Sol:- $\frac{\cancel{1+x^2}}{\cancel{1+x^2}} \frac{dy}{dx} + \frac{y}{1+x^2} = \frac{e^{\tan^{-1}x}}{1+x^2}$

$$\frac{dy}{dx} + \frac{1}{1+x^2} y = \frac{e^{\tan^{-1}x}}{1+x^2} \longrightarrow ①$$

where $p = \frac{1}{1+x^2}$, $Q = \frac{e^{\tan^{-1}x}}{1+x^2}$

I.F $e^{\int p(x)\, dx} = e^{\int \frac{1}{1+x^2} dx}$

$$= e^{\tan^{-1}x}$$

Now the solution of equ① is

$$y \cdot e^{\tan^{-1}x} = \int \frac{e^{\tan^{-1}x}}{1+x^2} \cdot e^{\tan^{-1}x}\, dx + c$$

put $\tan^{-1}x = t$

$\frac{1}{1+x^2} dx = dt$

$$= \int e^{t} \cdot e^{t}\, dt + c$$

$$= \int e^{2t}\, dt + c$$

$$= \frac{e^{2t}}{2} + c$$

$$y \cdot e^{\tan^{-1}x} = \frac{e^{2\tan^{-1}x}}{2} + c$$

(10) $e^{-y}\sec^2 y\, dy = dx + x\, dy$

Sol:-

$$e^{-y}\sec^2 y\, dy - x \cdot dy = dx$$

$$(e^{-y}\sec^2 y - x)\, dy = dx$$

$$e^{-y}\sec^2 y - x = \frac{dx}{dy}$$

$$\frac{dx}{dy} + x = e^{-y} \cdot \sec^2 y \rightarrow ①$$

where $P = 1$, $Q = e^{-y}\sec^2 y$

$$\text{I.F } e^{\int P(y)dy} = e^{\int (1) dy}$$

$$= e^{y}$$

Now the solution of equ ① is

$$x \cdot e^{y} = \int e^{-y} \cdot \sec^2 y \cdot e^{y}\, dy + c$$

$$= \int \sec^2 y\, dy + c$$

$$x \cdot e^{y} = \tan y + c.$$

(11) $y \cdot e^{y}\, dx = (y^2 - 2x e^{y})\, dy.$

Sol:-

$$y \cdot e^{y}\, dx = (y^2 - 2xe^{y})\, dy$$

$$\frac{dx}{dy} = \frac{y^2}{y \cdot e^{y}} - \frac{2x \cdot e^{y}}{y \cdot e^{y}}$$

$$\frac{dx}{dy} = \frac{y}{e^{y}} - \frac{2x}{y}$$

$$\frac{dx}{dy} + \left(\frac{2}{y}\right) x = \frac{y}{e^{y}} \rightarrow ①$$

where $P = \frac{2}{y}$, $Q = \frac{y}{e^{y}}$

I.F $e^{\int P(y)dy} = e^{\int \frac{2}{y} dy}$

$= e^{2\log y}$

$= e^{\log_e y^2}$

$= y^2$

Now, the solution of equ① is

$$x \cdot y^2 = \int \frac{y}{e^y} y^2 dy + c$$

$$= \int e^{-y} \cdot y^3 dy + c$$

D	I
$+ y^3$	e^{-y}
$- 3y^2$	$-e^{-y}$
$+ 6y$	e^{-y}
$- 6$	$-e^{-y}$
$+ 0$	e^{-y}

$$x \cdot y^2 = -y^3 e^{-y} - 3y^2 e^{-y} - 6y e^{-y} - 6e^{-y} + c$$

$$x \cdot y^2 = -e^{-y}\left[y^3 + 3y^2 + 6y + 6\right] + c.$$

Nill

(12) $(1+y^2) + (x - e^{-\tan^{-1} y})\frac{dy}{dx} = 0$

Sol:-
$$(x - e^{-\tan^{-1} y})\frac{dy}{dx} = -(1+y^2)$$

$$\frac{dy}{dx} = \frac{-(1+y^2)}{x - e^{-\tan^{-1} y}}$$

$$\frac{dx}{dy} = \frac{x}{-(1+y^2)} - \frac{e^{-\tan^{-1} y}}{-(1+y^2)}$$

$$\frac{dx}{dy} + \frac{1}{1+y^2} \cdot x = +\frac{e^{-\tan^{-1} y}}{1+y^2} \longrightarrow ①$$

where $P = \frac{1}{1+y^2}$ and $Q = \frac{e^{-\tan^{-1} y}}{1+y^2}$

I.F $e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y}$

Now the solution of equ ① is

$$x \cdot e^{Tan^{-1}y} = \int \frac{\cancel{e^{-Tan^{-1}y}}}{1+y^2} \cdot \cancel{e^{Tan^{-1}y}}\, dy + c$$

$$= \int \frac{1}{1+y^2}\, dy + c$$

$$x \cdot e^{Tan^{-1}y} = Tan^{-1}y + c.$$

(14) $\sqrt{1-y^2}\, dx = (\sin^{-1}y - x)\, dy$

Sol:-

$$\frac{dx}{dy} = \frac{\sin^{-1}y - x}{\sqrt{1-y^2}}$$

$$\frac{dx}{dy} = \frac{\sin^{-1}y}{\sqrt{1-y^2}} - \frac{x}{\sqrt{1-y^2}}$$

$$\frac{dx}{dy} + \frac{1}{\sqrt{1-y^2}} \cdot x = \frac{\sin^{-1}y}{\sqrt{1-y^2}} \rightarrow ①$$

Where $p = \frac{1}{\sqrt{1-y^2}}$ and $Q = \frac{\sin^{-1}y}{\sqrt{1-y^2}}$

$$I.F\ e^{\int p(y)dy} = e^{\int \frac{1}{\sqrt{1-y^2}} dy}$$

$$= e^{\cancel{Tan}\, \sin^{-1}y}$$

Now the solution of equ ① is

$$x \cdot e^{\sin^{-1}y} = \int \frac{\sin^{-1}y}{\sqrt{1-y^2}} \cdot \cancel{\sin}\, e^{\sin^{-1}y}\, dy + c$$

put $\sin^{-1}y = t$

$$\frac{1}{\sqrt{1-y^2}}\, dy = dt$$

$$= \int t \cdot e^t\, dt + c$$

$$= e^t \cdot t - e^t + c$$

$$= e^t(t-1) + c$$

$$x \cdot e^{\sin^{-1}y} = e^{\sin^{-1}y}(\sin^{-1}y - 1) + c$$

(19) $dr + (2r\cot\theta + \sin 2\theta)\, d\theta = 0$

Sol:- $(2r\cot\theta + \sin 2\theta)\, d\theta = -dr$

$$\frac{d\theta}{dr} = \frac{-1}{(2r\cot\theta + \sin 2\theta)}$$

$$\frac{dr}{d\theta} = -(2r\cot\theta + \sin 2\theta)$$

$$\frac{dr}{d\theta} + 2r\cot\theta = -\sin 2\theta$$

$$\frac{dr}{d\theta} + (2\cot\theta)r = -\sin 2\theta \rightarrow ①$$

where $P = 2\cot\theta$ and $Q = -\sin 2\theta$

$$I.F\ e^{\int P(\theta)d\theta} = e^{\int 2\cot\theta\, d\theta}$$

$$= e^{2\int \cot\theta\, d\theta}$$

$$= e^{2\log|\sin\theta|}$$

$$= e^{\log_e(\sin\theta)^2}$$

$$= \sin^2\theta$$

Now the solution of equ ① is

$$r.\sin^2\theta = \int -\sin 2\theta \cdot \sin^2\theta \cdot d\theta + c$$

$$= -\int 2\sin\theta \cdot \cos\theta \cdot \sin^2\theta \cdot d\theta + c$$

$$\boxed{\begin{array}{l}\sin\theta = t \\ \cos\theta . d\theta = dt\end{array}}$$

$$= -2\int \sin^3\theta \cdot \cos\theta \cdot d\theta + c$$

$$= -2\int t^3 . dt + c$$

$$= -\cancel{2} \cdot \frac{t^4}{\cancel{4}_2} + c$$

$$\boxed{r.\sin^2\theta = -\frac{\sin^4\theta}{2} + c}$$

(20) $\cosh x \frac{dy}{dx} + y\sinh x = 2\cosh^2 x \cdot \sinh x$.

Sol:-
$$\frac{\cosh x}{\cosh x} \cdot \frac{dy}{dx} + y \cdot \frac{\sinh x}{\cosh x} = \frac{2 \cdot \cosh^{\cancel{2}} x \cdot \sinh x}{\cancel{\cosh x}}$$

$$\frac{dy}{dx} + \tanh x \cdot y = 2 \cdot \sinh x \cdot \cosh x \quad \rightarrow ①$$

Here $P = \tanh x$ and $Q = 2\sinh x \cosh x$

I.F $e^{\int P(x)dx} = e^{\int \tanh x\, dx}$

$$= e^{\log_e |\operatorname{sech} x|}$$

$$= \underline{\operatorname{sech} x}.$$

Now the solution of equ ① is

$$y \cdot \operatorname{sech} x = \int 2\sinh x \cdot \cancel{\cosh x} \cdot \cancel{\operatorname{sech} x}\, dx + C$$

$$= 2\int \sinh x \cdot dx + C$$

$$\boxed{y \cdot \operatorname{sech} x = 2 \cdot \cosh x + C}$$

(16) $\frac{dy}{dx} + y \cdot \cot x = 4x \operatorname{cosec} x$ if $y = 0$ when $x = \pi/2$

Sol:-
$$\frac{dy}{dx} + y \cdot \cot x = 4x \cdot \operatorname{cosec} x$$

$$\frac{dy}{dx} + \cot x \cdot y = 4x \cdot \operatorname{cosec} x \quad \rightarrow ①$$

Here $P = \cot x$ and $Q = 4x \cdot \operatorname{cosec} x$.

I.F $e^{\int P(x)dx} = e^{\int \cot x \cdot dx}$

$$= e^{\log_e |\sin x|}$$

$$= \underline{\sin x}$$

Now the solution of equ① is

$$y \cdot \sin x = \int 4x \cdot \cancel{\operatorname{cosec} x} \cdot \cancel{\sin x} \cdot dx + C$$

$$= 4\int x \cdot dx + C$$

$$y \cdot \sin x = \cancel{4}^2 \cdot \frac{x^2}{\cancel{2}} + C$$

$$y \cdot \sin x = 2x^2 + C$$

$$(0) \cdot \sin \pi/2 = \cancel{2} \frac{\pi^2}{\cancel{4}_2} + C$$

$$0 = \frac{\pi^2}{2} + c$$

$$\boxed{\therefore c = -\pi^2/2}$$

(17) $\frac{dy}{dx} - y \cdot \tan x = 3 \cdot e^{-\sin x}$ if $y = 4$ when $x = 0$.

Sol:- $\frac{dy}{dx} + (-\tan x) \cdot y = 3 \cdot e^{-\sin x} \longrightarrow ①$

Here $P = -\tan x$ and $Q = 3 \cdot e^{-\sin x}$

I.F $e^{\int p(x)\,dx} = e^{-\int \tan x\,dx}$

$= e^{-(-\log |\cos x|)}$

$= e^{\log_e |\cos x|}$

$= \cos x$

Now the solution of equ ① is

$$y \cdot \cos x = \int 3 \cdot e^{-\sin x} \cdot \cos x\,dx + c$$

$$= 3 \int e^{-\sin x} \cdot \cos x\,dx + c \quad \text{put } \sin x = t, \; \cos x\,dx = dt$$

$$= 3 \cdot \int e^{-t}\,dt + c$$

$$= 3 \cdot e^{-t} (-1) + c$$

$$y \cdot \cos x = -3 \cdot e^{-\sin x} + c$$

$$4 \cdot (\cos 0) = -3\,e^{-\sin 0} + c$$

$$4 (1) = -3\,e^{0} + c$$

$$4 = -3(1) + c$$

$$c = 4 + 3$$

$$\boxed{c = 7}$$

(18) $\frac{dy}{dx} + y \cot x = 5 \cdot e^{\cos x}$ if $y = -4$ when $x = \pi/2$

Sol:- $\frac{dy}{dx} + \cot x \cdot y = 5 \cdot e^{\cos x} \longrightarrow ①$

Here $P = \cot x$ and $Q = 5 \cdot e^{\cos x}$

I.F $e^{\int p(x)\,dx} = e^{\int \cot x\,dx}$

$= e^{\log_e |\sin x|}$

$= \sin x$

Now the solution of equ(1) is

$$y \cdot \sin x = \int 5 \cdot e^{\cos x} \cdot \sin x \cdot dx + c$$

$$y \cdot \sin x = 5\int e^{\cos x} \cdot \sin x \, dx + c$$

put $\cos x = t$
$-\sin x \, dx = dt$
$\sin x \cdot dx = -dt$

$$= 5\int e^{t} (-dt) + c$$

$$= -5\int e^{t} dt + c$$

$$= -5e^{t} + c$$

$$y \cdot \sin x = -5 \cdot e^{\cos x} + c$$

$$(-4) \sin \pi/2 = -5 \cdot e^{\cos \pi/2} + c$$

$$(-4)(1) = -5 \cdot e^{0} + c$$

$$(-4) = -5(1) + c$$

$$c = -4 + 5$$

$$\boxed{c = 1} \Rightarrow y \cdot \sin x = -5e^{\cos x} + 1$$

(1) $x(1-x^2) \frac{dy}{dx} + (2x^2-1) y = x^3$.

Sol:- $x(1-x^2) \frac{dy}{dx} + (2x^2-1) y = x^3$

put $x = \cos t$
$dx = -\sin t \cdot dt$

$$\cos t (1-\cos^2 t) \frac{dy}{dx} + (2\cos^2 t - 1) y = \cos^3 t$$

$$\frac{\cancel{\cos t \cdot \sin^2 t}}{\cancel{\cos t \cdot \sin^2 t}} \frac{dy}{dx} + \frac{\cos 2t}{\cos t \cdot \sin^2 t} \cdot y = \frac{\cos^{3} t}{\cos t \cdot \sin^2 t}$$

$$\frac{dy}{dx} + \frac{\cos 2t}{\cos t \cdot \sin^2 t} \cdot y = \cot^2 t.$$

$$\cos t \sin^2 t \cdot \frac{dy}{-\sin t \cdot dt} + \cos 2t \cdot y = \cos^3 t$$

$$\frac{\cancel{-\cos t \cdot \sin t}}{\cancel{\cos t \cdot \sin t}} \frac{dy}{dt} + \frac{\cos 2t}{\cos t \cdot \sin t} \cdot y = \frac{\cos^2 t}{\cos t \cdot \sin t}$$

$$\frac{dy}{dt} + \left(\frac{-\cos 2t}{\cos t \cdot \sin t}\right) y = \frac{-\cos^2 t}{\sin t} \longrightarrow (1)$$

Here $P = \frac{-\cos 2t}{\cos t \cdot \sin t}$ and $Q = \frac{-\cos^2 t}{\sin t}$

$$\text{I.F } e^{\int p(t) dt} = e^{\int \frac{-\cos 2t}{\cos t \cdot \sin t} \cdot dt}$$

$$= e^{-\int \frac{2\cos 2t}{\sin 2t} \cdot dt}$$

$$= e^{-\log |\sin 2t|}$$

$$= e^{\log_e (\sin 2t)^{-1}} = (\sin 2t)^{-1} = \frac{1}{\sin 2t}$$

Now the solution of equ ① is

$$y \cdot \frac{1}{\sin 2t} = \int \frac{-\cos^2 t}{\sin t} \cdot \frac{1}{\sin 2t}\, dt + c$$

$$\frac{y}{\sin 2t} = -\int \frac{\cos^2 t}{\sin t \cdot 2\sin t \cdot \cos t}\, dt + c$$

$$\frac{y}{\sin 2t} = -\frac{1}{2}\int \operatorname{cosec} t \cdot \cot t \cdot dt + c$$

$$\frac{y}{\sin 2t} = -\frac{1}{2}(-\operatorname{cosec} t) + c$$

$$\frac{y}{\sin 2t} = \frac{\operatorname{cosec} t}{2} + c$$

$$\boxed{\frac{y}{\sin 2(\cos^{-1} x)} = \frac{\operatorname{cosec}(\cos^{-1} x)}{2} + c.}$$

(15) $x\left(\frac{dy}{dx} + y\right) = 1 - y$

Sol :- $x\left(\frac{dy}{dx} + y\right) = 1 - y$

$$\frac{dy}{dx} + y = \frac{1-y}{x}$$

$$\frac{dy}{dx} + y = \frac{1}{x} - \frac{y}{x}$$

$$\frac{dy}{dx} + y + \frac{y}{x} = \frac{1}{x}$$

$$\frac{dy}{dx} + \left(1 + \frac{1}{x}\right) y = \frac{1}{x} \rightarrow ①$$

Here $P = 1 + \frac{1}{x}$ and $Q = \frac{1}{x}$

I.F $e^{\int \left(1+\frac{1}{x}\right) dx} = e^{\int (1) dx + \int \frac{1}{x} dx}$

$$= e^{x + \log x}$$

$$= e^x \cdot e^{\log_e x}$$

$$= x \cdot e^x$$

Now the solution of equ ① is

$$y \cdot x \cdot e^x = \int \frac{1}{\cancel{x}} \cdot \cancel{x} \cdot e^x\, dx + c$$

$$= \int e^x\, dx + c$$

$$\boxed{x \cdot y \cdot e^x = e^x + c}$$

(9) $(1-x^2)\frac{dy}{dx} + 2xy = x\sqrt{1-x^2}$

Sol:- $\frac{1-x^2}{1-x^2}\cdot\frac{dy}{dx} + \frac{2xy}{1-x^2} = \frac{x\sqrt{1-x^2}}{1-x^2}$

$$\frac{dy}{dx} + \frac{2x}{1-x^2}\cdot y = \frac{x(1-x^2)^{1/2}}{(\sqrt{1-x^2})^2}$$

$$\frac{dy}{dx} + \frac{2x}{1-x^2}\cdot y = \frac{x}{\sqrt{1-x^2}} \rightarrow ①$$

Equ ① is of linear form $\frac{dy}{dx} + P.y = Q$.

Here $P = \frac{2x}{1-x^2}$ and $Q = \frac{x}{\sqrt{1-x^2}}$

I.F $e^{\int P(x)\,dx}$ $= e^{\int \frac{2x}{1-x^2}dx}$

$$= e^{-\int \frac{-2x}{1-x^2}dx}$$

$$= e^{-\log[1-x^2]}$$

$$= e^{\log_e (1-x^2)^{-1}}$$

$$= (1-x^2)^{-1}$$

$$= \frac{1}{1-x^2}$$

Now the solution of equ ① is

$$y\cdot\frac{1}{1-x^2} = \int \frac{x}{\sqrt{1-x^2}}\cdot\frac{1}{1-x^2}\,dx + C$$

$$= \int \frac{x}{(1-x^2)^{3/2}}\,dx + C$$

$$= \frac{-1}{2}\int \frac{-2x}{(1-x^2)^{3/2}}\,dx + C$$

put $1-x^2 = t$
$-2x.dx = dt$

$$= \frac{-1}{2}\int \frac{1}{t^{3/2}}\,dt + C.$$

$$= \frac{-1}{2}\int t^{-3/2}\,dt + C$$

$$= \frac{-1}{2}\,\frac{t^{-3/2+1}}{-3/2+1} + C$$

$$= \frac{+1}{\cancel{2}}\,\frac{t^{-1/2}}{+1/\cancel{2}} + C$$

$$= \frac{1}{t^{1/2}} + C$$

$$\frac{y}{1-x^2} = \frac{1}{\sqrt{1-x^2}} + C.$$

(1) $x(1-x^2)\frac{dy}{dx} + (3x^2-1)y = x^3$

Sol:- $\frac{dy}{dx} + \frac{3x^2-1}{x(1-x^2)} \cdot y = \frac{x^{\cancel{3}2}}{\cancel{x}(1-x^2)}$

$$\frac{dy}{dx} + \frac{3x^2-1}{x(1-x^2)} \cdot y = \frac{x^2}{1-x^2} \longrightarrow (1)$$

Here $P = \frac{3x^2-1}{x(1-x^2)}$ and $Q = \frac{x^2}{1-x^2}$

$$\text{I.F.} = e^{\int \frac{3x^2-1}{x(1-x^2)} dx} = e^{\int \frac{3x^2-1}{x-x^3} \cdot dx}$$

$$= e^{-\int \frac{1-3x^2}{x-x^3} dx} = e^{-\log|x-x^3|}$$

$$= \frac{1}{x(1-x^2)}$$

Now the solution of equ (1) is

$$y \cdot \frac{1}{x(1-x^2)} = \int \frac{x^{\cancel{2}}}{1-x^2} \cdot \frac{1}{\cancel{x}(1-x^2)} dx + C$$

$$= \frac{-1}{2}\int \frac{-2x}{(1-x^2)^2} dx + C$$

$$= \frac{-1}{2}\int \frac{1}{t^2} \cdot dt + C \qquad 1-x^2 = t, \quad -2x\,dx = dt$$

$$= \frac{-1}{2}\int t^{-2} dt + C$$

$$= \frac{+1}{2} \frac{t^{-1}}{-1} + C$$

$$= \frac{1}{2t} + C$$

$$\frac{y}{x(1-x^2)} = \frac{1}{2(1-x^2)} + C.$$

Reducible To The Linear Form:

(1) $\frac{dy}{dx} + x.\sin 2y = x^3.\cos^2 y$

Sol:- $\frac{dy}{dx} + x.\sin 2y = x^3.\cos^2 y$

$$\frac{1}{\cos^2 y}.\frac{dy}{dx} + \frac{x.\sin 2y}{\cos^2 y} = \frac{x^3.\cancel{\cos^2 y}}{\cancel{\cos^2 y}}$$

$$\sec^2 y.\frac{dy}{dx} + \frac{x.2\sin y.\cos y}{\cos^2 y} = x^3$$

$$\sec^2 y.\frac{dy}{dx} + 2x.\tan y = x^3$$

$\tan y = t$

$\sec^2 y.dy = dt$

$$\frac{dt}{dx} + 2x.t = x^3 \longrightarrow (1)$$

Here $P = 2x$. and $Q = x^3$

$$I.F \; e^{\int 2x.dx} = e^{2\int x.dx}$$

$$= e^{\cancel{2}.\frac{x^2}{\cancel{2}}}$$

$$= e^{x^2}$$

Now the solution of equ(1)

$$t.e^{x^2} = \int x^2.x e^{x^2}.dx + C$$

Let $x^2 = v$

$\cancel{3x^2 dx = dv}$

$2x.dx = dv$

$x.dx = \frac{dv}{2}$

$$= \int v.e^v.\frac{dv}{2} + C$$

$$= \frac{1}{2}\int v.e^v.dv + C$$

$$t.e^{x^2} = \frac{1}{2}e^v(v-1) + C$$

$$\tan y.e^{x^2} = \frac{1}{2}e^{x^2}(x^2-1) + C.$$

(2) $e^y \cdot y' = e^x(e^x - e^y)$

Sol:- $e^y \cdot \frac{dy}{dx} = e^x(e^x - e^y)$

$$e^y \cdot \frac{dy}{dx} = e^x \cdot e^x - e^x \cdot e^y$$

$$e^y \cdot \frac{dy}{dx} = e^{2x} - e^x \cdot e^y$$

$$e^y \frac{dy}{dx} + e^x e^y = e^{2x}$$

$e^y = t$

$e^y \cdot dy = dt$

$$\frac{dt}{dx} + e^x t = e^{2x} \longrightarrow ①$$

equ① is of the linear form di

$p = e^x$ and $Q = e^{2x}$

I.F $e^{\int p(x)dx} = e^{\int e^x \cdot dx}$

$$= e^{e^x}$$

Now the solution of ~~et~~ equ① is

$$t \cdot e^{e^x} = \int e^{2x} e^{e^x} \cdot dx + C$$

$$= \int e^x \cdot e^x \cdot e^{e^x} \cdot dx + C$$

Let $e^x = v$

$e^x dx = dv$

$$= \int v \cdot e^v dv + C$$

$$t \cdot e^{e^x} = e^v(v-1) + C$$

$$e^y \cdot e^{e^x} = e^{e^x}(e^x - 1) + C$$

(3) $(2x \log x - xy)\, dy = -2y\, dx$

Sol:- $\frac{dy}{dx} = \frac{-2y}{2x \cdot \log x - xy}$

$$\frac{dx}{dy} = \frac{2x \cdot \log x - xy}{-2y}$$

$$\frac{dx}{dy} = \frac{\cancel{2}x \cdot \log x}{-\cancel{2}y} + \frac{x\cancel{y}}{2\cancel{y}}$$

$$\frac{dx}{dy} = \frac{-x \cdot \log x}{y} + \frac{x}{2}$$

$$\cancel{\frac{dx}{dy} - \frac{x}{2} = \frac{-x \cdot \log x}{y}}$$

$$\frac{dx}{dy} + \frac{x \cdot \log x}{y} = \frac{x}{2}$$

$$\frac{1}{x} \cdot \frac{dx}{dy} + \frac{x \cdot \log x}{y} \cdot \frac{1}{x} = \frac{x}{2} \cdot \frac{1}{x}$$

$$\frac{dt}{dy} + \frac{1}{y} \cdot t = \frac{1}{2} \rightarrow (1)$$

put $\log x = t$

$\frac{1}{x} dx = dt$

Here $P = \frac{1}{y}$ and $Q = \frac{1}{2}$

I.F $e^{\int P(y) dy} = e^{\int \frac{1}{y} dy}$

$= e^{\log_e y}$

$= \underline{y}$

Now the solution of equ^n (1) is

$$t \cdot y = \int \frac{1}{2} \cdot y \cdot dy + C$$

$$t \cdot y = \frac{1}{2} \int y \cdot dy + c$$

$$= \frac{1}{2} \cdot \frac{y^2}{2} + c$$

$$t \cdot y = \frac{y^2}{4} + c$$

$$\log x \cdot y = \frac{y^2}{4} + c.$$

(4) $\frac{dy}{dx} - \tan x \cdot y = -y^2 \cdot \sec^2 x.$

Sol:- $\frac{dy}{dx} - \tan x \cdot y = -y^2 \sec^2 x$

$$\frac{1}{y^2} \frac{dy}{dx} - \frac{\tan x \cdot y}{y^2} = \frac{-y^2 \cdot \sec^2 x}{y^2}$$

$$\frac{1}{y^2} \cdot \frac{dy}{dx} - \frac{1}{y} \tan x = -\sec^2 x.$$

put $\frac{1}{y} = t$

$\frac{-1}{y^2} dy = dt$

$\frac{1}{y^2} \cdot dy = -dt$

$$\frac{-dt}{dx} + \tan x \cdot t = -\sec^2 x.$$

$$\frac{dt}{dx} - \tan x \cdot t = \sec^2 x \rightarrow (1)$$

Here $P = -\tan x$, $Q = \sec^2 x$.

I.F $e^{\int P(x) \cdot dx} = e^{-\int \tan x \cdot dx}$

$= e^{+\log_e (\cos x)}$

$= \underline{\cos x}$

Now the solution of equ① is

$$t \cdot \cos x = \int \sec^2 x \cdot \cos x \cdot dx + C$$

$$= \int \frac{1}{\cos^2 x} \cdot \cancel{\cos x \cdot} dx + C$$

$$t \cdot \cos x = \int \sec x \cdot dx + C$$

$$\frac{1}{y} \cdot \cos x = \log|\sec x + \tan x| + C$$

(5) $e^y\left(\frac{dy}{dx} + 1\right) = e^x$

Sol:- $e^y \cdot \frac{dy}{dx} + e^y = e^x$

put $e^y = t$

$e^y \cdot dy = dt$

$$\frac{dt}{dx} + (1)t = e^x \rightarrow ①$$

Here $P = 1$ and $Q = e^x$

I.F $e^{\int P(x)dx} = e^{\int (1)dx}$

$= e^x$

Now the solution of equ① is

$$t \cdot e^x = \int e^x \cdot e^x \cdot dx + C$$

$$= \int e^{2x} \cdot dx + C$$

$$t \cdot e^x = e^{2x} \cdot (2) + C$$

$$e^x \cdot e^y = 2 \cdot e^{2x} + C$$

(6) $(x+1) \cdot \frac{dy}{dx} + 1 = 2e^{-y}$

10

(7) $\tan y \cdot \frac{dy}{dx} + \tan x = \cos y \cdot \cos^2 x$.

Sol:-

$$\tan y \cdot \frac{dy}{dx} + \tan x = \cos y \cos^2 x$$

$$\frac{\tan y}{\cos y} \frac{dy}{dx} + \frac{\tan x}{\cos y} = \frac{\cos y \cdot \cos^2 x}{\cos y}$$

$$\sec y \cdot \tan y \cdot \frac{dy}{dx} + \sec y \cdot \tan x = \cos^2 x$$

$\sec y = t$

$\sec y \cdot \tan y \cdot dy = dt$

$$\frac{dt}{dx} + \tan x \cdot t = \cos^2 x \rightarrow ①$$

Here $P = \tan x$. $Q = \cos^2 x$

I.F $e^{\int P(x)dx} = e^{\int \tan x \cdot dx}$

$= e^{\log_e(\sec x)}$

$= \sec x$

Now the solution of equ① is

$$t \cdot \sec x = \int \cos^2 x \cdot \sec x \cdot dx + c$$

$$= \int \cos^2 x \cdot \frac{1}{\cos x} \cdot dx + c$$

$$= \int \cos x \cdot dx + c$$

$$t \cdot \sec x = \sin x + c$$

$$\sec x \cdot \sec y = \sin x + c$$

(8) $\frac{dz}{dx} + \frac{z}{x} \cdot \log z = \frac{z}{x} (\log z)^2$.

Sol:- $\frac{dz}{dx} + \frac{z}{x} \cdot \log z = \frac{z}{x} \cdot (\log z)^2$

$$\frac{1}{z \cdot (\log z)^2} \frac{dz}{dx} + \frac{1}{x} \frac{\cancel{z} \cdot \log z}{\cancel{z} \cdot (\log z)^2} = \frac{1}{x} \cdot \frac{\cancel{z} \cancel{(\log z)^2}}{\cancel{z} \cancel{(\log z)^2}}$$

$$\frac{1}{z (\log z)^2} \frac{dz}{dx} + \frac{1}{x} \frac{1}{\log z} = \frac{1}{x}$$

$\frac{1}{\log z} = t$

$\frac{-1}{(\log z)^2} \cdot \frac{1}{z} dz = dt$

$\frac{1}{z (\log z)^2} dz = -dt$

$$-\frac{dt}{dx} + \frac{1}{x} \cdot t = \frac{1}{x}$$

$$\frac{dt}{dx} - \frac{1}{x} \cdot t = \frac{-1}{x} \rightarrow ①$$

Here $P = \frac{-1}{x}$ and $Q = \frac{-1}{x}$.

I.F $e^{\int P(x) dx} = e^{\int -\frac{1}{x} dx}$

$$= e^{-\int \frac{1}{x} dx}$$

$$= e^{-\log x} = e^{\log_e x^{-1}}$$

$$\cancel{= \frac{1}{x}} = \frac{1}{x}$$

Now the solution of equ① is

$$t \cdot \frac{1}{x} = \int \frac{-1}{x} \cdot \frac{1}{x} \cdot dx + c$$

$$= -\int \frac{1}{x^2} dx + c$$

$$= -\int x^{-2} dx + c$$

$$= -\frac{x^{-1}}{-1} + c$$

$$t \cdot \frac{1}{x} = \frac{1}{x} + c$$

$$\frac{1}{x \cdot \log z} = \frac{1}{x} + c.$$

(8) $(x+1)\frac{dy}{dx}+1 = 2e^{-y}$.

sol:- $(x+1)\frac{dy}{dx} = 2e^{-y}-1$

$$\frac{dy}{dx} = \frac{2e^{-y}}{x+1} - \frac{1}{x+1}$$

$$\frac{dy}{dx}+\frac{1}{x+1} = \frac{2e^{-y}}{x+1}$$

$$\frac{1}{e^{-y}}\frac{dy}{dx}+\frac{1}{x+1}\cdot\frac{1}{e^{-y}} = \frac{2\cancel{e^{-y}}}{x+1}\times\frac{1}{\cancel{e^{-y}}}$$

$$e^{y}\cdot\frac{dy}{dx}+\frac{1}{x+1}\cdot e^{y} = \frac{2}{x+1}$$ put $e^{y}=t$

$e^{y}.dy=dt$

$$\frac{dt}{dx}+\frac{1}{x+1}\cdot t = \frac{2}{x+1} \longrightarrow ①$$

Here $P=\frac{1}{x+1}$ and $Q=\frac{1}{x+1}$

I.F $e^{\int p(x)dx} = e^{\int\frac{1}{x+1}\cdot dx}$

$= e^{\log_e(x+1)}$

$= x+1$.

Now the solution of equ^{n}① is

$$t\cdot(x+1) = \int\frac{2}{\cancel{x+1}}\cdot\cancel{(x+1)}\,dx + c$$

$= 2\int(1)\,dx + c$

$t\cdot(x+1) = 2x+C$

$e^{y}\cdot(x+1) = 2x+C$.

(9) $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)\,e^{x}\cdot\sec y$

sol:- $$\frac{dy}{dx}-\frac{\tan y}{1+x} = (1+x)\cdot e^{x}\cdot\sec y$$

$$\frac{1}{\sec y}\frac{dy}{dx}-\frac{\tan y}{1+x}\cdot\frac{1}{\sec y} = \frac{(1+x)e^{x}\cdot\cancel{\sec y}}{\cancel{\sec y}}$$

$$\cos y\cdot\frac{dy}{dx}-\frac{1}{1+x}\cdot\frac{\sin y}{\cancel{\cos y}}\cdot\cancel{\cos y} = (1+x)e^{x}$$

$$\cos y\cdot\frac{dy}{dx}-\frac{1}{1+x}\cdot\sin y = (1+x)e^{x}$$

$\sin y = t$

$$\frac{dt}{dx}-\frac{1}{1+x}\cdot t = (1+x)e^{x} \longrightarrow ①$$ $\cos y.dy=dt$

~~Equ~~ Equ① is of linear form

where $P=\frac{-1}{1+x}$ and $Q=(1+x)\,e^{x}$.

$$\text{I.F} \quad e^{\int p(x)dx} = e^{\int -\frac{1}{1+x}\cdot dx}$$

$$= e^{-\int \frac{1}{1+x}\cdot dx}$$

$$= e^{-\log(1+x)}$$

$$= e^{\log_e (1+x)^{-1}}$$

$$= \frac{1}{1+x}$$

Now the solution of equ① is

$$t\cdot \frac{1}{1+x} = \int \cancel{(1+x)}\cdot e^x \frac{1}{\cancel{1+x}}\cdot dx + C$$

$$= \int e^x \cdot dx + C$$

$$t\cdot \frac{1}{1+x} = e^x + C$$

$$\frac{\sin y}{1+x} = e^x + C$$

(10) $\frac{dy}{dx} + \frac{y\cdot \log y}{x} = \frac{y(\log y)^2}{x^2}$

Sol:-

$$\frac{dy}{dx} + \frac{y\cdot\log y}{x} = \frac{y\cdot(\log y)^2}{x^2}$$

$$\frac{1}{y\cdot(\log y)^2}\cdot\frac{dy}{dx} + \frac{\cancel{y}\cdot \cancel{\log y}}{x}\cdot\frac{1}{\cancel{y}\cdot(\log y)^{\cancel{2}}} = \frac{\cancel{y}\cdot\cancel{(\log y)^2}}{x^2}\cdot\frac{1}{\cancel{y}\,\cancel{(\log y)^2}}$$

$$\frac{1}{y(\log y)^2}\cdot\frac{dy}{dx} + \frac{1}{x}\cdot\frac{1}{\log y} = \frac{1}{x^2}$$

$$\frac{1}{\log y} = t$$

$$\frac{-1}{(\log y)^2}\cdot\frac{1}{y}\cdot dy = dt$$

$$\frac{1}{y\cdot(\log y)^2}\cdot dy = -dt$$

$$\frac{-dt}{dx} + \frac{1}{x}\cdot t = \frac{1}{x^2}$$

$$\frac{dt}{dx} - \frac{1}{x}\cdot t = \frac{-1}{x^2} \rightarrow ①$$

Here $P = \frac{-1}{x}$ and $Q = \frac{-1}{x^2}$

$$\text{I.F} \quad e^{\int -\frac{1}{x}\cdot dx} = e^{-\int \frac{1}{x}\cdot dx}$$

$$= e^{-\log(x)}$$

$$= e^{\log_e (x)^{-1}}$$

$$= \frac{1}{x}$$

Now the solution of equⁿ① is

$$t\cdot\frac{1}{x} = \int -\frac{1}{x^2}\cdot\frac{1}{x}\cdot dx + C$$

$$= \int \frac{-1}{x^3}\cdot dx + C$$

$$= -\int x^{-3}\,dx + c$$

$$= -\frac{x^{-2}}{-2} + c$$

$$t \cdot \frac{1}{x} = \frac{1}{2x^2} + c$$

$$\frac{1}{x \cdot \log y} = \frac{1}{2x^2} + c$$

Monday 16/09

Bernoulli's Equation:

(2) $(xy^2 - e^{1/x^3})\,dx - x^2y\,dy = 0$

sol:- $(xy^2 - e^{1/x^3})\,dx = x^2y\,dy$

$$\frac{dy}{dx} = \frac{xy^2 - e^{1/x^3}}{x^2y}$$

$$\frac{dy}{dx} = \frac{\cancel{x}y^{\cancel{2}}}{x^{\cancel{2}}\cancel{y}} - \frac{e^{1/x^3}}{x^2y}$$

$$\frac{dy}{dx} = \frac{y}{x} - \frac{e^{1/x^3}}{x^2y}$$

$$\frac{dy}{dx} - \frac{1}{x}\cdot y = -\frac{e^{1/x^3}}{x^2}y^{-1} \longrightarrow (1)$$

Equ(1) is of Bernoulli's form $\frac{dy}{dx} + P \cdot y = Q \cdot y^n$.

This can be reduced to linear form.

$$y\frac{dy}{dx} - \frac{1}{x}\cdot y\cdot y = \frac{-e^{1/x^3}}{x^2}\cdot \cancel{y^{-1}}\cdot \cancel{y}$$

$$y\cdot\frac{dy}{dx} - \frac{1}{x}\cdot y^2 = \frac{-e^{1/x^3}}{x^2}$$

$y^2 = t$

$2y\,dy = dt$

$y\,dy = dt\left(\frac{1}{2}\right)$

$$\frac{1}{2}\cdot\frac{dt}{dx} - \frac{1}{x}\cdot t = -\frac{e^{1/x^3}}{x^2}$$

$$\frac{dt}{dx} - \frac{2}{x}\cdot t = \frac{-2\cdot e^{1/x^3}}{x^2} \longrightarrow (2)$$

Equ(2) is in linear form. where $P = \frac{-2}{x}$ and $Q = \frac{-2e^{1/x^3}}{x^2}$.

$$\text{I.F } e^{\int P(x)dx} = e^{-2\int \frac{1}{x}dx}$$

$$= e^{-2\log(x)}$$

$$= e^{\log_e (x)^{-2}}$$

$$= \frac{1}{x^2}$$

Now the solution of equ(2) is

$$t \cdot \frac{1}{x^2} = \int \frac{-2e^{1/x^3}}{x^2} \cdot \frac{1}{x^2} \cdot dx + c$$

$$= -2\int e^{1/x^3} \cdot \frac{1}{x^4} \, dx + c$$

$$= -2\int e^{x^{-3}} \cdot x^{-4} \, dx + c$$

$x^{-3} = v$

$-3 \cdot x^{-3-1} dx = dv$

$x^{-4} \cdot dx = \frac{-1}{3} dv$

$$= -2\int e^v \cdot \frac{-1}{3} dv + c$$

$$= \frac{2}{3}\int e^v \cdot dv + c$$

$$t \cdot \frac{1}{x^2} = \frac{2}{3} e^v + c$$

$$y^2 \cdot \frac{1}{x^2} = \frac{2}{3} e^{x^{-3}} + c$$

$$\frac{y^2}{x^2} = \frac{2}{3} e^{1/x^3} + c.$$

(1) $x \cdot \frac{dy}{dx} + y = x^3 y^6$

Sol:-

$$x \cdot \frac{dy}{dx} + y = x^3 y^6$$

$$\frac{x}{y^6} \cdot \frac{dy}{dx} + \frac{\cancel{y}}{y^{\cancel{6}\,5}} = \frac{x^3 \cancel{y^6}}{\cancel{y^6}}$$

$$x \cdot y^{-6} \cdot \frac{dy}{dx} + y^{-5} = x^3$$

$y^{-5} = t$

$-5y^{-6} dy = dt$

$y^{-6} dy = \frac{-1}{5} dt$

$$\cancel{x} \cdot y^{-6} \frac{dy}{dx} + \frac{1}{x} \cdot y^{-5} = x^2$$

$$\frac{-1}{5} \frac{dt}{dx} + \frac{1}{x} \cdot t = x^2$$

$$\frac{dt}{dx} - \frac{5}{x} \cdot t = -5x^2 \longrightarrow (1)$$

Equ(1) is in linear form.

where $P = \frac{-5}{x}$ and $Q = -5x^2$

$$I.F \; e^{\int P(x) dx} = e^{\int \frac{-5}{x} dx}$$

$$= e^{-5\int \frac{1}{x} dx}$$

$$= e^{-5 \log x}$$

$$= e^{\log_e (x)^{-5}}$$

$$= x^{-5}$$

$$= \frac{1}{x^5}$$

Now the solution of equ ① is

$$t \cdot \frac{1}{x^5} = \int -5x^2 \cdot \frac{1}{x^5}\, dx + c$$

$$= -5 \int x^{-3} \cdot dx + c$$

$$= -5 \frac{x^{-2}}{-2} + c$$

$$t \cdot \frac{1}{x^5} = \frac{5}{2} \cdot \frac{1}{x^2} + c.$$

$$\frac{1}{x^5 \cdot y^5} = \frac{5}{2} \cdot \frac{1}{x^2} + c.$$

(3) $xy(1 + xy^2) \cdot \frac{dy}{dx} = 1$

Sol:- $xy(1 + xy^2) = \frac{dx}{dy}.$

$$\frac{dx}{dy} = xy + x^2y^3$$

$$\frac{dx}{dy} - xy = x^2y^3$$

$$\frac{dx}{dy} - y \cdot x = x^2y^3 \to ①$$

Equ ① is of Bernoulli's form $\frac{dx}{dy} + P \cdot x = Q \cdot x^n$

This can be reduced to linear form.

$$\frac{dx}{dy} - y \cdot x = x^2 \cdot y^3$$

$$\frac{1}{x^2} \cdot \frac{dx}{dy} - y \cdot \frac{x}{x^2} = \frac{x^2 \cdot y^3}{x^2}$$

$$\frac{1}{x^2} \cdot \frac{dx}{dy} - y\left(\frac{1}{x}\right) = y^3$$

$-\frac{1}{x} = t$

$$\frac{dt}{dy} + y \cdot t = -y^3 \to ②$$

$+\left(\frac{-1}{x^2}\right) dx = dt$

$\frac{1}{x^2} \cdot dx = dt$

Equ② is in linear form.

where $P = y$ and $Q = -y^3$

I.F $e^{\int P(y)dy} = e^{\int y \cdot dy}$

$$= e^{y^2/2}$$

Now the solution of equ ② is

$$t \cdot e^{y^2/2} = \int -y^3 \cdot e^{y^2/2} dy + c.$$

$$= -\int y\cdot y^2\cdot e^{y^2/2}\cdot dy + c \qquad \frac{y^2}{2} = v \rightarrow \boxed{y^2 = 2v}$$

$$= -\int e^{v}\cdot 2v\cdot dv + c \qquad \frac{1}{\cancel{2}}\cdot\cancel{2}y\,dy = dv$$

$$= -2\int e^{v}\cdot v\cdot dv + c \qquad y\cdot dy = dv$$

$$t\cdot e^{y^2/2} = -2\cdot e^{v}(v-1) + c$$

$$\frac{1}{x}\cdot e^{y^2/2} = -2\cdot e^{y^2/2}\left(\frac{y^2}{2} - 1\right) + c.$$

(5) $\frac{dy}{dx} - x^2 y = y^2\cdot e^{-x^3/3}$

Sol:- $\frac{dy}{dx} - x^2\cdot y = e^{-x^3/3}\cdot y^2 \rightarrow ①$

Equⁿ ① is of Bernoulli's form $\frac{dy}{dx} + P\cdot y = Q\cdot y^n$.

∴ This can be reduced to linear form.

$$\frac{1}{y^2}\frac{dy}{dx} - x^2\cdot \cancel{y}\,\frac{1}{y^{\cancel{2}}} = e^{-x^3/3}\cdot \cancel{y^2}\,\cancel{\frac{1}{y^2}}$$

$$\frac{1}{y^2}\frac{dy}{dx} - x^2\cdot\frac{1}{y} = e^{-x^3/3} \qquad \frac{1}{y} = t$$

$$-\frac{dt}{dx} - x^2\cdot t = e^{-x^3/3} \qquad -\frac{1}{y^2}\cdot dy = dt$$

$$\frac{dt}{dx} + x^2\cdot t = -e^{-x^3/3} \rightarrow ② \qquad \frac{1}{y^2}\,dy = -dt$$

Equⁿ ② is in linear form.

where $P = x^2$ and $Q = -e^{-x^3/3}$

$$I.F\ e^{\int P(x)dx} = e^{\int x^2\cdot dx}$$

$$= e^{x^3/3}.$$

Now the solution of equⁿ ② is

$$t\cdot e^{x^3/3} = \int -\cancel{e^{-x^3/3}}\cdot\cancel{e^{x^3/3}}\,dx + c$$

$$= -1\int (1)\,dx + c$$

$$t\cdot e^{x^3/3} = -x + c$$

$$\frac{1}{y}\cdot e^{x^3/3} = -x + c$$

(4) $2.\frac{dy}{dx} = \frac{y}{x} + \frac{y^2}{x^2}$.

Sol:-

$$2.\frac{dy}{dx} = \frac{y}{x} + \frac{y^2}{x^2}$$

$$2.\frac{dy}{dx} - \frac{y}{x} = \frac{y^2}{x^2} \rightarrow ①$$

Equ^n ① is of Bernoulli's form $\frac{dy}{dx} + Py = Q.y^n$. This can be reduced to linear form.

$$\frac{2}{y^2}.\frac{dy}{dx} - \frac{1}{x}.\frac{y}{y^2} = \frac{1}{x^2}.\frac{y^2}{y^2}$$

$$\frac{2}{y^2}.\frac{dy}{dx} - \frac{1}{x}.\frac{1}{y} = \frac{1}{x^2}$$

put $\frac{1}{y} = t$

$$-\frac{1}{y^2}.dy = dt$$

$$\frac{1}{y^2}dy = -dt$$

$$2.\frac{-dt}{dx} - \frac{1}{x}.t = \frac{1}{x^2}$$

$$\frac{dt}{dx} + \frac{1}{2x}.t = \frac{-1}{2x^2} \rightarrow ②$$

~~Equ^{n}① is of Bernoulli's form $\frac{dy}{dx} + P.y = Q.y^n$~~

~~Eq this can be reduced to linear form.~~

Equ^{n}② is in linear form.

Where $P = \frac{1}{2x}$ and $Q = \frac{-1}{2x^2}$

I.F $e^{\int P(x)dx}$

$$= e^{\frac{1}{2}\int \frac{1}{x}dx}$$

$$= e^{\frac{1}{2}\log x}$$

$$= e^{\log_e (x)^{1/2}}$$

$$= x^{1/2}$$

Now the solution of equ^{n}② is

$$t.x^{1/2} = \int \frac{-1}{2x^2}.\frac{1}{2x}dx + c$$

$$= \frac{-1}{4}\int \frac{1}{x^3}dx + c$$

$$= \frac{-1}{4}\int x^{-3}.dx + c$$

$$= \frac{-1}{4}\frac{x^{-2}}{-2} + c$$

$$t.x^{1/2} = \frac{1}{8x^2} + c$$

$$\frac{1}{y}.x^{1/2} = \frac{1}{8x^2} + c.$$

$$t.x^{1/2} = \int \frac{-1}{2x^2}.x^{1/2}dx + c$$

$$= \frac{-1}{2}\int x^{-2}.x^{1/2}dx + c$$

$$= \frac{-1}{2}\int x^{-3/2}dx + c$$

$$= \frac{-1}{2}\frac{x^{-1/2}}{-1/2} + c$$

$$t.x^{1/2} = \frac{1}{x^{1/2}} + c$$

$$\frac{1}{y}.x^{1/2} = \frac{1}{x^{1/2}} + c.$$

(6) $(x^3y^2+xy)\,dx = dy$

Sol:- $(x^3y^2+xy)\,dx = dy$

$$\frac{dy}{dx} = x^3y^2+xy$$

$$\frac{dy}{dx} - xy = x^3y^2 \rightarrow ①$$

Equ^n ① is of Bernoulli's form $\frac{dy}{dx}+P\cdot y = Q y^n$.

This can be reduced to linear form.

$$\frac{1}{y^2}\frac{dy}{dx} - x\cdot \cancel{y}\cdot\frac{1}{y^{\cancel{2}}} = x^3\cdot\frac{\cancel{y^2}}{\cancel{y^2}}$$

$$\frac{1}{y^2}\cdot\frac{dy}{dx} - x\cdot\frac{1}{y} = x^3$$

$$-\frac{dt}{dx} - x\cdot t = x^3 \qquad \frac{1}{y}=t$$

$$\qquad \frac{-1}{y^2}dy = dt$$

$$\frac{dt}{dx} + x\cdot t = -x^3 \rightarrow ② \qquad \frac{1}{y^2}dy = dt\,(-1)$$

Equation ② is in linear form

where $P = x$ and $Q = -x^3$

I.F $e^{\int P(x)dx} = e^{\int x\cdot dx}$

$$= e^{x^2/2}$$

Now the solution of equ^n ② is

$$t\cdot e^{x^2/2} = \int -x^3\cdot e^{x^2/2}\,dx + c \qquad \Rightarrow \boxed{x^2 = 2v}$$

$$= -\int x^2\cdot x\cdot e^{x^2/2}\,dx + c \qquad \frac{x^2}{2} = v$$

$$\qquad \frac{1}{\cancel{2}}\cdot\cancel{2}x\,dx = dv$$

$$= -\int 2v\cdot e^v\cdot dv + c \qquad x\,dx = dv$$

$$= -2\int e^v\cdot v\,dv + c$$

$$t\cdot e^{x^2/2} = -2\cdot e^v(v-1) + c$$

$$\frac{1}{y}\cdot e^{x^2/2} = -2\cdot e^{x^2/2}\left(\frac{x^2}{2}-1\right) + c.$$

(7) $\frac{dy}{dx} + y = xy^3$

Sol:- $\frac{dy}{dx} + y = xy^3 \rightarrow (1)$

Equ^n (1) is of linear form $\frac{dy}{dx} + P.y = Q y^n$.

This can be reduced to linear form.

$\frac{1}{y^3}\frac{dy}{dx} + \cancel{y}\frac{1}{y^{\cancel{3}}_2} = x.\cancel{y^3}\frac{1}{\cancel{y^3}}$

$\frac{1}{y^3}.\frac{dy}{dx} + \frac{1}{y^2} = x.$

$y^{-3}.\frac{dy}{dx} + y^{-2} = x$

Put $y^{-2} = t$

$-2.y^{-3}\,dy = dt$

$y^{-3}.dy = \frac{-1}{2}dt$

$\frac{-1}{2}.\frac{dt}{dx} + t = x.$

$\frac{dt}{dx} - 2t = -2x \rightarrow (2)$

Equ^n (2) is in linear form

where $P = -2$ and $Q = -2x$

I.F $e^{\int P(x)dx}$

$= e^{\int -2.dx}$

$= e^{-2\int (1)dx}$

$= e^{-2.x} \quad \underline{= e^{-2x}}$

Now the solution of equ^n(2) is

$t.e^{-2x} = \int -2x.e^{-2x}dx + c$

$= +\int v.e^{v}.\frac{-1}{2}dv + c$

$-2x = v$

$-2(1)dx = dv$

$dx = \frac{-1}{2}.dv$

$= \frac{-1}{2}\int e^{v}.v.dv + c$

$= \frac{-1}{2}.e^{v}(v-1) + c$

$t.e^{-2x} = \frac{-1}{2}.e^{-2x}(-2x-1) + c$

$\frac{1}{y^2}.e^{-2x} = \frac{1}{2}.e^{-2x}(2x+1) + c$

(8) $\frac{dy}{dx} + y \cdot \tan x = y^3 \cdot \cos x.$

Sol:- $$\frac{dy}{dx} + y \cdot \tan x = \cos x \cdot y^3 \rightarrow ①$$

Equn ① is of Bernoulli's form $\frac{dy}{dx} + P \cdot y = Q\, y^n$.

This can be reduced to linear form

$$\frac{1}{y^3}\frac{dy}{dx} + \cancel{y} \cdot \tan x \cdot \frac{1}{y^{\cancel{3}2}} = \cos x \cdot \cancel{y^3}\,\frac{1}{\cancel{y^3}}$$

$$y^{-3} \cdot \frac{dy}{dx} + \tan x \cdot y^{-2} = \cos x.$$

put $y^{-2} = t$

$-2y^{-3} dy = dt$

$y^{-3} dy = -\frac{1}{2} dt$

$$-\frac{1}{2}\frac{dt}{dx} + \tan x \cdot t = \cos x$$

$$\frac{dt}{dx} - 2 \cdot \tan x \cdot t = -2 \cdot \cos x$$

Equn ② is in linear form.

where $P = -2\tan x$. and $Q = -2\cos x$

I.F $e^{\int P(x) dx}$
$$= e^{\int -2\tan x \cdot dx}$$
$$= e^{-2\int \tan x\, dx}$$
$$= e^{+2 \log(\cos x)}$$
$$= e^{\log_e (\cos x)^2}$$
$$= \cos^2 x.$$

Now the solution of equn ② is.

$$t \cdot \cos^2 x = \int -2\cos x \cdot \cos^2 x\, dx + c$$
$$= -2\int \cos^3 x\, dx + c$$
$$= \cancel{-2 \cdot \frac{\cos^4 x}{4}}$$
$$= \frac{-2}{4}\int (\cos 3x + 3\cos x)\, dx + c$$
$$= \frac{-1}{2}\left[\frac{\sin 3x}{3} + 3\sin x\right] + c.$$

(9) $\frac{dy}{dx} + \frac{x}{1-x^2}\cdot y = x\sqrt{y}$.

Sol:- $\frac{dy}{dx} + \frac{x}{1-x^2}\cdot y = x\sqrt{y}$

$$\frac{dy}{dx} + \frac{x}{1-x^2}\cdot y = x\cdot y^{1/2} \rightarrow (1)$$

Equⁿ (1) is of Bernoulli's form $\frac{dy}{dx} + P\cdot y = Q\cdot y^n$

This can be reduced to linear form.

$$\frac{1}{y^{1/2}}\frac{dy}{dx} + \frac{x}{1-x^2}\cdot y\cdot\frac{1}{y^{1/2}} = x\cdot \cancel{y^{1/2}\cdot\frac{1}{y^{1/2}}}$$

$$\frac{1}{y^{1/2}}\cdot\frac{dy}{dx} + \frac{x}{1-x^2}\cdot y\cdot y^{-1/2} = x.$$

$$\frac{1}{y^{1/2}}\cdot\frac{dy}{dx} + \frac{x}{1-x^2}y^{1/2} = x.$$

$y^{1/2} = t$

$\frac{1}{2}\cdot y^{\frac{1}{2}-1}\,dy = dt$

$\frac{1}{2}\cdot y^{-1/2}\cdot dy = dt.$

$\frac{1}{y^{1/2}}\cdot dy = 2\cdot dt$

$$2\frac{dt}{dx} + \frac{x}{1-x^2}\cdot t = x \rightarrow (2)$$

$$\frac{dt}{dx} + \frac{x}{2(1-x^2)}\cdot t = \frac{x}{2} \rightarrow (2)$$

Equⁿ (2) is in linear form.

where $P = \frac{x}{2(1-x^2)}$ and $Q = \frac{x}{2}$

$$I.F\ e^{\int P(x)\,dx}$$

$$= e^{\int\frac{x}{2(1-x^2)}dx}$$

$$= e^{\frac{1}{2}\int\frac{x}{1-x^2}\,dx}$$

$$= e^{\frac{1}{2}\times\frac{-1}{2}\int\frac{-2x}{1-x^2}\,dx}$$

$$= e^{-1/4\cdot\log(1-x^2)}$$

$$= e^{\log(1-x^2)^{-1/4}}$$

$$= e^{\log_e(1-x^2)^{-1/4}}$$

$$= (1-x^2)^{-1/4} = \frac{1}{(1-x^2)^{1/4}}$$

Now the solution of equⁿ (2) is

$$t\cdot\frac{1}{(1-x^2)^{1/4}} = \int\frac{x}{2}\cdot(1-x^2)^{-1/4}\cdot dx + c$$

$$= \frac{1}{2}\int x\cdot(1-x^2)^{-1/4}\,dx + c$$

$$= \frac{1}{2(-2)}\int(-2x)\,(1-x^2)^{-1/4}\,dx + c$$

$$= \frac{-1}{4}\int v^{-1/4}.dv + c \qquad 1-x^2 = v$$
$$-2x.dx = dv$$
$$= \frac{-1}{4} \cdot \frac{v^{-1/4+1}}{-1/4+1} + c$$
$$= \frac{-1}{4} \frac{v^{3/4}}{3/4} + c$$
$$t.(1-x^2)^{-1/4} = \frac{-1}{3} \cdot v^{3/4} + c$$
$$y^{1/2}(1-x^2)^{-1/4} = \frac{-1}{3} \cdot (1-x^2)^{3/4} + c.$$

(10) $y - \cos x \cdot \frac{dy}{dx} = y^2(1-\sin x)\cos x$. G.T $y=2$ when $x=0$.

sol:-
$$y - \cos x \cdot \frac{dy}{dx} = y^2(1-\sin x)\cos x$$
$$-\cos x \frac{dy}{dx} = y^2(1-\sin x)\cos x - y$$
$$\frac{-\cancel{\cos x}}{-\cancel{\cos x}} \frac{dy}{dx} = \frac{y^2(1-\sin x)\cancel{\cos x}}{-\cancel{\cos x}} - \frac{y}{-\cos x}$$
$$\frac{dy}{dx} = -y^2(1-\sin x) + \frac{y}{\cos x}$$
$$\frac{dy}{dx} - \sec x \cdot y = y^2(\sin x - 1) \rightarrow (1)$$

Equ^n(1) is of Bernoulli's form $\frac{dy}{dx} + Py = Q \cdot y^n$

This can be reduced to linear form.
$$\frac{1}{y^2}\frac{dy}{dx} - \sec x \cdot \frac{y}{y^2} = \frac{\cancel{y^2}(\sin x - 1)}{\cancel{y^2}}$$
$$\frac{1}{y^2}\frac{dy}{dx} - \sec x \cdot \frac{1}{y} = \sin x - 1$$

$\frac{1}{y} = t$

$\frac{-1}{y^2}dy = dt$

$\frac{1}{y^2}dy = -dt$

$$\frac{-dt}{dx} - \sec x \cdot t = \sin x - 1$$
$$\frac{dt}{dx} + \sec x \cdot t = 1 - \sin x \rightarrow (2)$$

Equ^n(2) is in linear form.

Where $P = \sec x$ and $Q = 1 - \sin x$

$$\text{I.F } e^{\int P(x)dx} = e^{\int \sec x\, dx}$$
$$= e^{\log_e(\sec x + \tan x)}$$
$$= \sec x + \tan x$$

Now the solution of equ^n ② is

$$t.(\sec x + \tan x) = \int (1-\sin x)(\sec x + \tan x)\,dx + C$$

$$= \int (\cancel{\sec x + \tan x + \sin x.\sec x - \sin x.\tan x})\,dx + C.$$

$$= \int \sec x.dx + \int \tan x.dx - \int \cancel{\tan x.dx}$$

$$= \int (1-\sin x)\left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right) dx + C$$

$$= \int (1-\sin x)\left(\frac{1+\sin x}{\cos x}\right) dx + C$$

$$= \int \frac{1-\sin^2 x}{\cos x}\,dx + C$$

$$= \int \frac{\cos^2 x}{\cos x}\,dx + C$$

$$t.(\sec x + \tan x) = \sin x + C$$

$$\frac{1}{y}(\sec x + \tan x) = \sin x + C.$$

Given that $y=2$ when $x=0$

$$\frac{1}{2}(\sec 0^\circ + \tan 0^\circ) = \sin 0^\circ + C$$

$$\frac{1}{2}(1+0) = 0 + C$$

$$\frac{1}{2}(1) = C$$

$$\boxed{\therefore C = 1/2}$$

$$\therefore \frac{1}{y}(\sec x + \tan x) = \sin x + 1/2$$

(11) $\frac{dy}{dx} - \tan x.\, y = -y^2.\sec x.$

Sol:- $\frac{dy}{dx} - \tan x.\, y = -y^2.\sec x \rightarrow ①$ is Bernoulli's.

$$\frac{1}{y^2}.\frac{dy}{dx} - \tan x.y.\frac{1}{y^2} = \frac{-y^2 \sec x}{y^2}$$

$$\frac{1}{y^2}\frac{dy}{dx} - \tan x.\frac{1}{y} = -\sec x.$$

$\frac{1}{y} = t$

$\frac{-1}{y^2}dy = dt$

$\frac{1}{y^2}dy = -dt$

$$-\frac{dt}{dx} - \tan x.t = -\sec x$$

$$\frac{dt}{dx} + \tan x.t = +\sec x. \rightarrow ②$$

Equ^n ② is in linear form.

where $P = \tan x$ and $Q = +\sec x$.

$$I.F \quad e^{\int P(x)dx} = e^{\int \tan x \, dx}$$

$$= e^{\log_e (\sec x)}$$

$$= \sec x$$

Now the solution of equ^n (2) is

$$t \cdot \sec x = \int + \sec x \cdot \sec x \, dx + c$$

$$= + \int \sec^2 x \, dx + c$$

$$t \cdot \sec x = + \tan x + c$$

$$\frac{1}{y} \cdot \sec x = + \tan x + c.$$

Tuesday
17/09

Exact Differential Equations:

(2) $[\cos x \tan y + \cos(x+y)] dx + [\sin x \sec^2 y + \cos(x+y)] dy = 0$

Sol:- $[\cos x \tan y + \cos(x+y)] dx + [\sin x \cdot \sec^2 y + \cos(x+y)] dy = 0 \longrightarrow (1)$

Equ^n (1) is of exact form of $M dx + N dy = 0$.

where $M = \cos x \cdot \tan y + \cos(x+y)$

and $N = \sin x \sec^2 y + \cos(x+y)$

$$M = \cos x \cdot \tan y + \cos x \cos y - \sin x \sin y$$

$$\left(\frac{\partial M}{\partial y}\right)_{x = \text{const}} = \cos x \cdot \sec^2 y + \cos x(-\sin y) - \sin x (\cos y)$$

$$\frac{\partial M}{\partial y} = \cos x \cdot \sec^2 y - \cos x \sin y - \sin x \cos y$$

$$N = \sin x \cdot \sec^2 y + \cos x \cos y - \sin x \sin y$$

$$\left(\frac{\partial N}{\partial x}\right)_{y = \text{const}} = \sec^2 y \cos x + \cos y(-\sin x) - \sin y \cos x$$

$$\frac{\partial N}{\partial x} = \sec^2 y \cos x - \cos y \sin x - \sin y \cos x$$

$$= \cos x \cdot \sec^2 y - \cos x \sin y - \sin x \cos y$$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

Hence Equ^n ① is an exact.

Solution of equ^n ① is $\int M\,dx + \int N\,dy = c$.

$$\int [\cos x \tan y + \cos(x+y)]dx + \int [\sin x \sec^2 y + \cos(x+y)]dy = C$$

$$\int \cos x \tan y\, dx + \int (\cos x \cos y - \sin x \sin y)\,dx$$

$$+ \int \sin x \sec^2 y\, dy + \int (\cos x \cdot \cos y - \sin x \sin y)\,dy = C$$

$$\int \cos x \tan y\, dx + \int \cos x \cos y\, dx - \int \sin x \sin y\, dx$$

$$+ \int \sin x \sec^2 y\, dy + \int \cos x \cos y\, dy - \int \sin x \sin y\, dy = C$$

$$\tan y \int \cos x\, dx + \cos y \int \cos x\, dx - \sin y \int \sin x\, dx + 0 + 0 - 0 = C$$

$$\tan y \sin x + \cos y \sin x - \sin y(-\cos x) = C$$

$$\tan y \sin x + \sin x \cos y + \cos x \sin y = C$$

$$\sin x \tan y + \sin(x+y) = C$$

(5) $(1+e^{x/y})\,dx + \left(1-\frac{x}{y}\right)e^{x/y}\,dy = 0$

Sol:- $(1+e^{x/y})\,dx + \left(1-\frac{x}{y}\right)e^{x/y}\,dy = 0 \longrightarrow ①$

$M = 1+e^{x/y}$ and $N = \left(1-\frac{x}{y}\right)e^{x/y}$

$$\frac{\partial M}{\partial y} = 0 + e^{x/y}\cdot\frac{-x}{y^2}$$

$$= -e^{x/y}\cdot\frac{x}{y^2}$$

$$\frac{\partial N}{\partial x} = \left(0-\frac{1}{y}\right)e^{x/y} + e^{x/y}\frac{d}{dx}\left(\frac{x}{y}\right)\left(1-\frac{x}{y}\right)$$

$$= \frac{-1}{y}e^{x/y} + e^{x/y}\frac{1}{y}\cdot\left(1-\frac{x}{y}\right)$$

$$= \cancel{\frac{-1}{y}e^{x/y}} + \cancel{\frac{1}{y}e^{x/y}} - \frac{x}{y^2}e^{x/y}$$

$$= -e^{x/y}\cdot\frac{x}{y^2}$$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

Hence Equ^n ① is an exact.

Now the solution of equ^n ① is $\int M\,dx + \int N\,dy = C$

$$\int (1+e^{x/y})\,dx + \int (1-x/y)\,e^{x/y}dy = C$$

$$\int (1)\,dx + \int e^{x/y}\,dx + \int (0)\,dy - \int \frac{x}{y}\cdot e^{x/y}\,dy = C$$

$$x + \frac{e^{x/y}}{1/y} + 0 - 0 = c$$

$$x + y \cdot e^{x/y} = c.$$

(6) $(\sec x \tan x \tan y - e^x)\,dx + \sec x \cdot \sec^2 y\, dy = 0 \longrightarrow (1)$

sol:- Equ^n (1) is of exact ~~form~~ differential equation.

$$M\,dx + N\,dy = 0.$$

where $M = \sec x \tan x \tan y - e^x$

$$\frac{\partial M}{\partial y}_{(x=const)} = \sec x \cdot \tan x \cdot \sec^2 y - 0.$$
$$= \sec x \cdot \tan x \cdot \sec^2 y$$

and $N = \sec x \cdot \sec^2 y.$

$$\frac{\partial N}{\partial x}_{(y\,const)} = \sec^2 y \cdot \sec x \cdot \tan x$$
$$= \sec x \cdot \tan x \sec^2 y.$$

$$\therefore \boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

Hence Equ^n(1) is ~~of~~ an exact form.

Now the solution of equ^n(1) is $\int M\,dx + \int N\,dy = c$

$$\int (\sec x \tan x \tan y - e^x)\,dx + \int \sec x \sec^2 y\, dy = c$$

$$\tan y \int \sec x \tan x \cdot dx - \int e^x dx + 0 = c$$

$$\tan y \sec x - e^x = c.$$

(7) $(5x^4 + 3x^2y^2 - 2xy^3)\,dx + (2x^3y - 3x^2y^2 - 5y^4)\,dy = 0. \longrightarrow (1)$

sol:- Equ^n (1) is of exact differential equation of $M\,dx + N\,dy = 0.$

where $M = 5x^4 + 3x^2y^2 - 2xy^3$ and $N = 2x^3y - 3x^2y^2 - 5y^4$

$$\frac{\partial M}{\partial y}_{(x=con)} = 0 + 3x^2(2y) - 2x \cdot 3y^2$$
$$= 6x^2y - 6xy^2$$

$$\frac{\partial N}{\partial x}_{(y=const)} = 2y(3x^2) - 3y^2(2x) - 0$$
$$= 6x^2y - 6xy^2$$

$$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

Hence Equ^n(1) is an exact.

Solution of equ^n(1) is $\int M\,dx + \int N\,dy = c$

$$\int(5x^4+3x^2y^2-2xy^3)\,dx+\int(2x^3y-3x^2y^2-5y^4)\,dy=c$$

$$5\int x^4.dx+3y^2\int x^2.dx-2y^3\int x.dx+\int 2x^3y.dy-\int 3x^2y^2dy-\int 5y^4.dy=c$$

$$\cancel{5}\left(\frac{x^5}{\cancel{5}}\right)+\cancel{3}y^2\left(\frac{x^3}{\cancel{3}}\right)-\cancel{2}y^3\left(\frac{x^2}{\cancel{2}}\right)+0-0-\cancel{5}\cdot\frac{y^5}{\cancel{5}}=c$$

$$x^5+x^3y^2-x^2y^3-y^5=c.$$

$$x^5-y^5+x^3y^2-x^2y^3=c.$$

(3) $\dfrac{dy}{dx}+\dfrac{y\cos x+\sin y+y}{\sin x+x\cos y+x}=0$

Sol:- $$\frac{dy}{dx}=-\frac{y\cos x+\sin y+y}{\sin x+x\cdot\cos y+x}$$

$$(\sin x+x\cos y+x)\,dy=-(y\cos x+\sin y+y)\,dx$$

$$(y\cos x+\sin y+y)\,dx+(\sin x+x\cos y+x)\,dy=0 \longrightarrow (1)$$

Equ^n (1) is of exact differential equation of $Mdx+Ndy=0$

where $M=y\cos x+\sin y+y$ and $N=\sin x+x\cos y+x$.

$$\frac{\partial M}{\partial y}=\cos x.(1)+\cos y+1 \qquad \frac{\partial N}{\partial x}=\cos x+\cos y(1)+1$$

$$=\cos x+\cos y+1 \qquad =\cos x+\cos y+1$$

$$\boxed{\therefore \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}}$$

Hence Equ^n (1) is an exact.

Now the solution of equ^n (1) is $\int Mdx+\int Ndy=c$.

$$\int(y\cos x+\sin y+y)\,dx+\int(\sin x+x\cos y+x)\,dy=c$$

$$y\int\cos x.dx+\sin y\int dx+y\int dx+\int\sin x\,dy+\int x\cdot\cos y\,dy+\int x\,dy=c$$

$$y\sin x+\sin y\cdot x+y\cdot x+0+0+0=c$$

$$\sin x.y+x.\sin y+xy=c$$

(4) $(2x^3 - xy^2 - 2y + 3)dx - (x^2y + 2x)dy = 0 \rightarrow ①$

Sol:- Equⁿ ① is of an exact differential equation of $Mdx + Ndy = 0$

where $M = 2x^3 - xy^2 - 2y + 3$ and $N = -x^2y - 2x$.

$$\frac{\partial M}{\partial y} = 0 - x\cdot 2y - 2 + 0 = -2xy - 2.$$

$$\frac{\partial N}{\partial x} = -y\cdot(2x) - 2 = -2xy - 2.$$

$$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

Hence Equⁿ ① is an exact.

Now the solution of equⁿ ① is $\int M dx + \int N dy = C$

$$\int(2x^3 - xy^2 - 2y + 3)dx + \int(-x^2y^2 - 2x)dy = C.$$

$$2\int x^3 dx - y^2\int x\,dx - \int 2y\,dx + 3\int(1)dx - \int x^2y^2 dy - \int 2x\,dy = C$$

$$2\cdot\frac{x^4}{4} - y^2\cdot\frac{x^2}{2} - 2yx + 3x - 0 - 0 = C$$

$$\frac{x^4}{2} - \frac{x^2}{2}y^2 - 2xy + 3x = C$$

$$\frac{x^2}{2}\left[x^2 - y^2\right] - 2xy + 3x = C$$

(7) $\left(\cos x \cdot \log(2y-8) + \frac{1}{x}\right)dx + \frac{\sin x}{y-4}dy = 0 \rightarrow ①$

Sol:- Equⁿ ① is of an exact differential form $Mdx + Ndy = 0$

where $M = \cos x\cdot\log(2y-8) + \frac{1}{x}$ and $N = \frac{\sin x}{y-4}$

$$\frac{\partial M}{\partial y} = \cos x\cdot\frac{1}{2y-8}(2-0) + 0 = \frac{\cos x}{y-4}$$

$$\frac{\partial N}{\partial x} = \frac{1}{y-4}(\cos x) = \frac{\cos x}{y-4}$$

$$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

Hence Equⁿ ① is an exact.

Now the solution of equⁿ ① is $\int M dx + \int N dy = C$.

$$\int\left(\cos x\cdot\log(2y-8) + \frac{1}{x}\right)dx + \int\frac{\sin x}{y-4}dy = C.$$

$$\log(2y-8)\int\cos x\,dx + \int\frac{1}{x}dx + 0 = C$$

$$\log(2y-8)\cdot\sin x + \log x = C$$

(8) $(2xy\cos x^2 - 2xy + 1)dx + (\sin x^2 - x^2)dy = 0 \rightarrow ①$

Sol: Equn ① is of an exact differential equation

$$Mdx + Ndy = 0$$

where $M = 2xy\cos x^2 - 2xy + 1$ and $N = \sin x^2 - x^2$

$$\frac{\partial M}{\partial y} = 2x\cos x^2 - 2x + 0 \qquad \frac{\partial N}{\partial x} = \cos x^2(2x) - 2x$$

$$= 2x(\cos x^2 - 1) \qquad = 2x(\cos x^2 - 1)$$

$$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

Hence Equn ① is an exact.

Now the solution of equn ① is $\int M dx + \int N dy = C$

$$\int(2xy\cos x^2 - 2xy + 1)dx + \int(\sin x^2 - x^2)dy = C$$

$$y\int 2x\cdot\cos x^2 dx - 2y\int x\,dx + \int(1)dx + \int \sin x^2 dy - \int x^2 \cdot dy = C$$

$$y\int \cos t \cdot dt - \cancel{2}y\frac{x^2}{\cancel{2}} + x + 0 - 0 = C$$

put $x^2 = t$
$2x dx = dt$.

$$y\cdot\sin t - x^2 y + x = C$$

$$\sin x^2 \cdot y - x^2 y + x = C$$

$$y(\sin x^2 - x^2) + x = C$$

(9) $(y^2 e^{xy^2} + 4x^3)dx + (2xy e^{xy^2} - 3y^2)dy = 0 \rightarrow ①$

Sol:- Equn ① is of an exact differential equation

$$Mdx + Ndy = 0$$

where

$M = y^2 e^{xy^2} + 4x^3$ and $N = 2xye^{xy^2} - 3y^2$

$$\frac{\partial M}{\partial y} = y^2 e^{xy^2}(2y)(x) + e^{xy^2}\cdot 2y \qquad \frac{\partial N}{\partial x} = 2y[x\cdot e^{xy^2}(y^2)(1) + e^{xy^2}(1)] - 0$$

$$= 2y[xy^2\cdot e^{xy^2} + e^{xy^2}] \qquad = 2y[xy^2 e^{xy^2} + e^{xy^2}]$$

$$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

Hence Equn ① is an exact.

Now the solution of equn ① is $\int M dx + \int N dy = C$.

$$\int(y^2 e^{xy^2} + 4x^3)dx + \int(2xy\cdot e^{xy^2} - 3y^2)dy = C$$

$$\int y^2 e^{xy^2} dx + \int 4x^3 dx + \int 2xye^{xy^2} dy - \int 3y^2 dy = C.$$

$$y^2\int e^{xy^2}dx + 4\int x^3 dx + 0 - 3\int y^2 dy = c$$

$$\cancel{y^2}\cdot\frac{e^{xy^2}}{\cancel{y^2}} + \cancel{4}\frac{x^4}{\cancel{4}} - \cancel{3}\frac{y^3}{\cancel{3}} = c$$

$$e^{xy^2} + x^4 - y^3 = c.$$

(10) $[y\cdot(1+\frac{1}{x}) + \cos y]\,dx + (x + \log x - x\sin y)\,dy = 0 \rightarrow (1)$

Sol:- Equ^n (1) is of an exact differential equation $Mdx + Ndy = 0$

where $M = y(1+\frac{1}{x}) + \cos y$ and $N = x + \log x - x\sin y$

$$\frac{\partial M}{\partial y} = (1+\frac{1}{x}) + (-\sin y)$$

$$= 1 + \frac{1}{x} - \sin y$$

$$\frac{\partial N}{\partial x} = 1 + \frac{1}{x} - \sin y\,(1)$$

$$= 1 + \frac{1}{x} - \sin y$$

$$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

Hence Equ^n (1) is an exact.

Now the solution of equ^n (1) is $\int M dx + \int N dy = c$.

$$\int [y(1+\frac{1}{x}) + \cos y]\,dx + \int (x + \log x - x\sin y)\,dy = c$$

$$y\int [(1+\frac{1}{x}) + \cos y]\,dx + \int x\,dy + \int \log x\,dy - \int x\sin y\,dy = c$$

$$y\int (1)dx + \int \frac{1}{x}dx + \cos y\int (1)dx + 0 + 0 - 0 = c$$

$$y\cdot x + \log x + \cos y\cdot x = c$$

$$xy + x\cdot\cos y + \log x = c$$

Non-Exact ! 19-09 (Thursday)

(Method - I)

(4) $(3xy^2 - y^3)\,dx - (2x^2y - xy^2)\,dy = 0 \longrightarrow ①$

Sol:- Equ^n ① is of exact form $Mdx + Ndy = 0$.

where $M = 3xy^2 - y^3$ and $N = -2x^2y + xy^2$.

$\frac{\partial M}{\partial y} = 3x(2y) - 3y^2$ $\qquad$ $\frac{\partial N}{\partial x} = -2(2x) + y^2$

$= 6xy - 3y^2$ $\qquad$ $= -4x + y^2$

$$\boxed{\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

Hence equ^n ① is non-exact.

Equ^n ① can be reduced to exact by multiplying an integrating factor.

→ clearly equ^n ① is homogeneous degree "3".

→ $Mx + Ny = (3xy^2 - y^3)x + (-2x^2y + xy^2)y$

$= 3x^2y^2 - xy^3 - 2x^2y^2 + xy^3$

$= x^2y^2 \neq 0.$

$$\boxed{\therefore Mx + Ny \neq 0}$$

$\therefore$ I.F $= \frac{1}{Mx+Ny} = \frac{1}{x^2y^2}$

from ①,

$(3xy^2 - y^3)\,dx - (2x^2y - xy^2)\,dy = 0$

$\frac{1}{x^2y^2}(3xy^2 - y^3)\,dx - \frac{(2x^2y - xy^2)\,dy}{x^2y^2} = 0 \times \frac{1}{x^2y^2}$

$\frac{y^2(3x - y)}{x^2y^2}\,dx - \frac{xy(2x-y)}{x^2y^2}\,dy = 0.$

$\frac{3x-y}{x^2}\,dx - \frac{2x-y}{xy}\,dy = 0$

$\left(\frac{3x}{x^2} - \frac{y}{x^2}\right)dx - \left(\frac{2x}{xy} - \frac{y}{xy}\right)dy = 0$

$\left(\frac{3}{x} - \frac{y}{x^2}\right)dx - \left(\frac{2}{y} - \frac{1}{x}\right)dy = 0 \longrightarrow ②$

Equ^n ② is of an exact form $Mdx + Ndy = 0$

where $M = \frac{3}{x} - \frac{y}{x^2}$ and $N = -\left(\frac{2}{y} - \frac{1}{x}\right)$

$M = \frac{3}{x} - \frac{y}{x^2}$ $\quad\quad$ $N = -\frac{2}{y} + \frac{1}{x}$

$\frac{\partial M}{\partial y} = 0 - \frac{1}{x^2}(1)$ $\quad\quad$ $\frac{\partial N}{\partial x} = -0 + \left(\frac{-1}{x^2}\right)$

$= \frac{-1}{x^2}$ $\quad\quad$ $= \frac{-1}{x^2}$.

$$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

Equⁿ (2) is an exact form.

Now the solution of equⁿ (2) is $\int M dx + \int N dy = c$.

$$\int\left(\frac{3}{x} - \frac{y}{x^2}\right)dx + \int\left(-\frac{2}{y} + \frac{1}{x}\right)dy = c$$

$$3\int\frac{1}{x}dx - y\int\frac{1}{x^2}dx - 2\int\frac{1}{y}dy + \int\frac{1}{x}dy = c$$

$$3\log x - y\int x^{-2}dx - 2\log y + 0 = c$$

$$3\log x - y\cdot\frac{x^{-1}}{-1} - 2\log y = c$$

$$3\log x + \frac{y}{x} - 2\log y = c.$$

$$\log x^3 + \frac{y}{x} - \log y^2 = c$$

$$\log\left(\frac{x^3}{y^2}\right) + \frac{y}{x} = c.$$

(5) $(x^2 - 3xy + 2y^2)\,dx + x(3x - 2y)\,dy = 0 \rightarrow (1)$

Sol: Equⁿ (1) is of an exact form $Mdx + Ndy = 0$

where $M = x^2 - 3xy + 2y^2$ and $N = 3x^2 - 2xy$.

$\frac{\partial M}{\partial y} = 0 - 3x(1) + 2(2y)$ $\quad\quad$ $\frac{\partial N}{\partial x} = 3(2x) - 2y(1)$

$= 4y - 3x$. $\quad\quad$ $= 6x - 2y$

$$\boxed{\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

Hence equⁿ (1) is non-exact.

Equⁿ (1) can be reduced to exact form by multiplying an Integrating factor.

→ clearly Equⁿ (1) is a homogeneous degree "2".

$$Mx + Ny = (x^2 - 3yx + 2y^2)x + (3x^2 - 2xy)y$$

$$= x^3 - \cancel{3x^2y} + \cancel{2xy^2} + \cancel{3x^2y} - \cancel{2xy^2}$$

$$= x^3 \neq 0.$$

$$\boxed{Mx + Ny \neq 0}$$

$$\therefore I.F = \frac{1}{Mx+Ny} = \frac{1}{x^3}.$$

from ①,

$$(x^2 - 3xy + 2y^2)\,dx + (3x^2 - 2xy)\,dy = 0$$

$$\frac{x^2 - 3xy + 2y^2}{x^3}\,dx + \frac{3x^2 - 2xy}{x^3}\,dy = 0$$

$$\left(\frac{x^2}{x^3} - \frac{3xy}{x^3} + \frac{2y^2}{x^3}\right)dx + \left(\frac{3x^2}{x^3} - \frac{2xy}{x^3}\right)dy = 0$$

$$\left(\frac{1}{x} - \frac{3y}{x^2} + \frac{2y^2}{x^3}\right)dx + \left(\frac{3}{x} - \frac{2y}{x^2}\right)dy = 0. \rightarrow ②$$

Equ^n ② is an exact form of $Mdx + Ndy = 0$.

where $M = \frac{1}{x} - \frac{3y}{x^2} + \frac{2y^2}{x^3}$ and $N = \frac{3}{x} - \frac{2y}{x^2}$

$$\frac{\partial M}{\partial y} = 0 - \frac{3}{x^2}(1) + \frac{2}{x^3}(2y) \qquad \frac{\partial N}{\partial x} = 3\left(\frac{-1}{x^2}\right) - 2y(-2)x^{-3}$$

$$= \frac{-3}{x^2} + \frac{4y}{x^3} \qquad = \frac{-3}{x^2} + 4\frac{y}{x^3}$$

$$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

Clearly ~~Hence~~ equ^n ② is an exact.

Now the solution of equ^n ② is $\int M dx + \int N dy = c$

$$\int\left(\frac{1}{x} - \frac{3y}{x^2} + \frac{2y^2}{x^3}\right)dx + \int\left(\frac{3}{x} - \frac{2y}{x^2}\right)dy = C$$

$$\int \frac{1}{x}dx - 3y\int x^{-2}dx + 2y^2\int x^{-3}dx + 0 = C$$

$$\log x - 3y\frac{x^{-1}}{-1} + \cancel{2}y^2\frac{x^{-2}}{-\cancel{2}} = C$$

$$\log x + 3\frac{y}{x} - \frac{y^2}{x^2} = C$$

(i) $(x^2y - 2xy^2)\,dx - (x^3 - 3x^2y)\,dy = 0. \rightarrow ①$

Sol:- Equ^n ① is an exact form of $Mdx + Ndy = 0$

Where $M = x^2y - 2xy^2$ and $N = -x^3 + 3x^2y$

$$\frac{\partial M}{\partial y} = x^2(1) - 2x(2y) \qquad \frac{\partial N}{\partial x} = -3x^2 + 3y(2x)$$

$$= x^2 - 4xy \qquad = -3x^2 + 6xy$$

$$\boxed{\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

Hence equ^n ① is non-exact.

Equ^n ① can be reduced to exact form by multiplying an integrating factor.

→ clearly equⁿ① is a homogeneous degree '3'

$$Mx+Ny = (x^2y-2xy^2)x - (x^3-3x^2y)y$$

$$= x^3\cancel{y} - 2x^2y^2 - x^3\cancel{y} + 3x^2y^2$$

$$= x^2y^2 \neq 0$$

$$\boxed{Mx+Ny \neq 0}$$

$$I.F = \frac{1}{Mx+Ny} = \frac{1}{x^2y^2}$$

$$\frac{(x^2y-2xy^2)}{x^2y^2}\cdot dx - \frac{(x^3-3x^2y)}{x^2y^2}dy = 0$$

$$\left(\frac{x^2y}{x^2y^2} - \frac{2xy^2}{x^2y^2}\right)dx - \left(\frac{x^3}{x^2y^2} - \frac{3x^2y}{x^2y^2}\right)dy = 0$$

$$\left(\frac{1}{y} - \frac{2}{x}\right)dx - \left(\frac{x}{y^2} - \frac{3}{y}\right)dy = 0 \quad \rightarrow ②$$

clea Equⁿ ② is an exact form $Mdx+Ndy=0$.

where $M = \frac{1}{y} - \frac{2}{x}$ and $N = -\frac{x}{y^2} + \frac{3}{y}$

$$\frac{\partial M}{\partial y} = -\frac{1}{y^2} - 0 \qquad \frac{\partial N}{\partial x} = -\frac{1}{y^2}(1) + 0$$

$$= -\frac{1}{y^2} \qquad = -\frac{1}{y^2}$$

$$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

clearly equⁿ② is an exact.

Now the solution of equⁿ② is $\int M dx + \int N dy = c$

$$\int\left(\frac{1}{y} - \frac{2}{x}\right)dx + \int\left(\frac{-x}{y^2} + \frac{3}{y}\right)dy = c$$

$$\frac{1}{y}\int(1)\,dx - 2\int\frac{1}{x}dx - \int\frac{x}{y^2}dy + 3\int\frac{1}{y}dy = c$$

$$\frac{1}{y}\cdot x - 2\cdot\log x - 0 + 3\log y = c$$

$$\frac{x}{y} - \log x^2 + \log y^3 = c$$

$$\log\frac{y^3}{x^2} + \frac{x}{y} = c$$

(2). $(xy-2y^2)\,dx - (x^2-3xy)\,dy = 0 \rightarrow ①$

sol:- Equn ① is ~~of~~ of an exact form $M\,dx + N\,dy = 0$.

where $M = xy - 2y^2$ and $N = -(x^2 - 3xy)$

$\frac{\partial M}{\partial y} = x.(1) - 2(2y)$ $\qquad$ $\frac{\partial N}{\partial x} = -[2x - 3y(1)]$

$= x - 4y$ $\qquad$ $= -2x + 3y$

$$\boxed{\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

Equn ① is non-exact.

Equn① can be reduced to exact by multiplying an integrating factor.

→ clearly equn① is a homogeneous degree '2'.

$Mx + Ny = (xy - 2y^2)x - (x^2 - 3xy)y$

$= x^2\cancel{y} - 2xy^2 - x^2\cancel{y} + 3xy^2$

$= xy^2 \neq 0.$

$$\boxed{Mx + Ny \neq 0}$$

$$I.F. = \frac{1}{Mx+Ny} = \frac{1}{xy^2}$$

from ①,

$$\frac{(xy-2y^2)}{xy^2}\,dx - \left(\frac{x^2-3xy}{xy^2}\right)dy = 0.$$

$$\left(\frac{\cancel{x}\cancel{y}}{\cancel{x}y^{\cancel{2}}} - \frac{2y^{\cancel{2}}}{xy^{\cancel{2}}}\right)dx - \left(\frac{x^{\cancel{2}}}{\cancel{x}y^2} - \frac{3\cancel{x}\cancel{y}}{\cancel{x}y^{\cancel{2}}}\right)dy = 0$$

$$\left(\frac{1}{y} - \frac{2}{x}\right)dx - \left(\frac{x}{y^2} - \frac{3}{y}\right)dy = 0 \rightarrow ②$$

Equn ② is an exact form of $M\,dx + N\,dy = 0$

where $M = \frac{1}{y} - \frac{2}{x}$ and $N = -\frac{x}{y^2} + \frac{3}{y}$

$\frac{\partial M}{\partial y} = \frac{-1}{y^2} - 0$ $\qquad$ $\frac{\partial N}{\partial x} = -\frac{1}{y^2}(1) + 0$

$= \frac{-1}{y^2}$ $\qquad$ $= \frac{-1}{y^2}$

$$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

~~So~~ clearly equn ② is an exact.

Now the solution of equⁿ ② is $\int M dx + \int N dy = c$

$$\int\left(\frac{1}{y} - \frac{2}{x}\right) dx + \int\left(-\frac{x}{y^2} + \frac{3}{y}\right) dy = c$$

$$\frac{1}{y}\int(1) dx - 2\int \frac{1}{x} dx + \int -\frac{x}{y^2} dy + 3\int \frac{1}{y} dy = c$$

$$\frac{1}{y}(x) - 2\log x + 0 + 3 \cdot \log y = c$$

$$\frac{x}{y} - \log x^2 + \log y^3 = c$$

$$\log\left(\frac{y^3}{x^2}\right) + \frac{x}{y} = c.$$

(3). $x^2y\, dx - (x^3+y^3)\, dy = 0 \rightarrow ①$

Sol:- Equⁿ① is of an exact form $M dx + N dy = 0$

Where $M = x^2y$ and $N = -(x^3+y^3)$

$$\frac{\partial M}{\partial y} = x^2(1) = x^2 \qquad \frac{\partial N}{\partial x} = -3x^2 + 0 = -3x^2$$

$$\boxed{\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

Hence equⁿ① is non-exact.

Equⁿ① can be reduced to exact by multiplying Integrating factor.

Clearly equⁿ① is homogeneous degree '3'.

$Mx + Ny = (x^2y)x + [-(x^3+y^3)]y$

$= x^2y \cdot x - x^3y - y^4$

$= x^3y - x^3y - y^4$

$= x^3(y-y) - y^4 \neq 0$ $y^4 \neq 0$

$$\boxed{Mx + Ny \neq 0}$$

$$I.F = \frac{1}{Mx+Ny} = \frac{1}{x^3(y-y) - y^4} \quad \frac{1}{y^4}$$

from①,

$$x^2y\, dx - (x^3+y^3)\, dy = 0.$$

$$\frac{x^2y}{x^3(y-y) - y^4} \quad \frac{x^2y}{y^4}\, dx - \frac{(x^3+y^3)}{y^4} dy = 0$$

$$\frac{x^2}{y^3} dx - \left(\frac{x^3}{y^4} + \frac{y^3}{y^4}\right) dy = 0$$

$$\frac{x^2}{y^3} \cdot dx - \left(\frac{x^3}{y^4} + \frac{1}{y}\right) dy = 0 \rightarrow ②$$

Equⁿ② is an exact.

where $M = \frac{x^2}{y^3}$ and $N = -\frac{x^3}{y^4} - \frac{1}{y}$.

$\frac{\partial M}{\partial y} = x^2 (-3) y^{-4}$ $\qquad$ $\frac{\partial N}{\partial x} = \frac{-1}{y^4} 3x^2 - 0$

$= \frac{-3x^2}{y^4}$ $\qquad$ $= \frac{-3x^2}{y^4}$

$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$ $\therefore$ Equ^n (2) is an exact.

Now the solution of equ^n (2) is $\int M dx + \int N dy = C$.

$$\int \frac{x^2}{y^3} dx + \int \left(\frac{-x^3}{y^4} - \frac{1}{y}\right) dy = C$$

$$\frac{1}{y^3} \int x^2 dx - \int \frac{x^3}{y^4} dy - \int \frac{1}{y} dy = C$$

$$\frac{1}{y^3} \frac{x^3}{3} - 0 - \log y = C$$

$$\frac{x^3}{3y^3} - \log y = C$$

Saturday
21/09/2019

Method - II.

(4) $(xy^2 + 2x^2y^3) dx + (x^2y - x^3y^2) dy = 0$. $\rightarrow$ (1)

Sol:- Equ^n (1) is of an exact form $Mdx + Ndy = 0$

where $M = xy^2 + 2x^2y^3$ and $N = x^2y - x^3y^2$

$\frac{\partial M}{\partial y} = x \cdot 2y + 2x^2 (3y^2)$ $\qquad$ $\frac{\partial N}{\partial x} = y(2x) - y^2 \cdot 3x^2$

$= 2xy + 6x^2y^2$ $\qquad$ $= 2xy - 3x^2y^2$

$\boxed{\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$

Hence equ^n (1) is non-exact.

Equ^n (1) can be reduced to exact by multiplying Integrating factor.

$\rightarrow$ clearly

from (1), $(xy^2 + 2x^2y^3) dx + (x^2y - x^3y^2) dy = 0$

$y(xy + 2x^2y^2) dx + x(xy - x^2y^2) dy = 0$

$Mx - Ny = xy(xy + 2x^2y^2) - xy \cdot (xy - x^2y^2)$

$= \cancel{x^2y^2} + 2x^3y^3 - \cancel{x^2y^2} + x^3y^3$

$= 3x^3y^3 \neq 0$

$I.F = \frac{1}{Mx - Ny}$

$= \frac{1}{3x^3y^3}$

from ①

$$(xy^2+2x^2y^3)\,dx+(x^2y-x^3y^2)\,dy=0$$

$$\left(\frac{xy^2+2x^2y^3}{3x^3y^3}\right)dx+\left(\frac{x^2y-x^3y^2}{3x^3y^3}\right)dy=0$$

$$\left(\frac{xy^2}{3x^3y^3}+\frac{2x^2y^3}{3x^3y^3}\right)dx+\left(\frac{x^2y}{3x^3y^3}-\frac{x^3y^2}{3x^3y^3}\right)dy=0$$

$$\left(\frac{1}{3x^2y}+\frac{2}{3x}\right)dx+\left(\frac{1}{3xy^2}-\frac{1}{3y}\right)dy=0 \rightarrow ②$$

Equn ② is an exact.

where $M=\frac{1}{3x^2y}+\frac{2}{3x}$ and $N=\frac{1}{3xy^2}-\frac{1}{3y}$

$$\frac{\partial M}{\partial y}=\frac{1}{3x^2}\left(\frac{-1}{y^2}\right)+0 \qquad \frac{\partial N}{\partial x}=\frac{1}{3y^2}\left(\frac{-1}{x^2}\right)-0$$

$$=\frac{-1}{3x^2y^2} \qquad =\frac{-1}{3x^2y^2}$$

$$\boxed{\therefore \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}}$$

Clearly Equn ② is an exact.

Now the solution of equn ② is $\int M\,dx+\int N\,dy=C$.

$$\int\left(\frac{1}{3x^2y}+\frac{2}{3x}\right)dx+\int\left(\frac{1}{3xy^2}-\frac{1}{3y}\right)dy=c$$

$$\frac{1}{3y}\int\frac{1}{x^2}dx+\frac{2}{3}\int\frac{1}{x}dx+\int\frac{1}{3y^2x}dy-\frac{1}{3}\int\frac{1}{y}dy=c$$

$$\frac{1}{3y}\frac{x^{-1}}{-1}+\frac{2}{3}\log x+0-\frac{1}{3}\log y=c$$

$$\frac{-1}{3xy}+\frac{2}{3}\log x-\frac{1}{3}\log y=c$$

$$\frac{-1}{xy}+2\log x-\log y=3c$$

$$\frac{-1}{xy}+\log x^2-\log y=3c$$

$$\frac{-1}{xy}+\log\left(\frac{x^2}{y}\right)=3c$$

$$-\frac{1}{3}\left[\frac{1}{xy}+\log\left(\frac{x^2}{y}\right)\right]=c$$

(6) $(xy \sin xy + \cos xy)\, y\, dx + (xy \sin xy - \cos xy)\, x \cdot dy = 0$ $\rightarrow (1)$

sol:- Equ^n(1) is an exact form $M dx + N dy = 0$

where $M = xy^2 \sin xy + y \cos xy$

$$\frac{\partial M}{\partial y} = x\left[2y \cdot \sin xy + y^2 \cos xy \cdot x\right] + y(\sin xy)(x) + \cos xy\,(1)$$

$$= 2xy \cdot \sin xy + x^2y^2 \cos xy - xy \sin xy + \cos xy$$

$$= xy \sin xy + x^2y^2 \cos xy + \cos xy.$$

and $N = x^2y \sin xy - \cos xy \cdot (x)$

$$\frac{\partial N}{\partial x} = y\left[2x \cdot \sin xy + x^2 \cos xy \cdot y\right] - \left[x \cdot (-\sin xy)\, y + \cos xy\,(1)\right]$$

$$= 2xy \sin xy + x^2y^2 \cos xy + xy \sin xy - \cos xy$$

$$= 3xy \sin xy + x^2y^2 \cos xy - \cos xy$$

$$\boxed{\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

Hence equ^n(1) is non-exact.

Equ^n(1) can be reduced to exact by multiplying Integrating factor.

from(1),

(1) $y(xy \sin xy + \cos xy)\, dx + x(xy \sin xy - \cos xy)\, dy = 0$

(2) $Mx - Ny = xy(xy \sin xy + \cos xy) - [xy(xy \sin xy - \cos xy)]$

$$= x^2y^2 \cancel{\sin xy} + xy \cos xy - x^2y^2 \cancel{\sin xy} + xy \cos xy$$

$$= 2xy \cos xy \neq 0$$

$$\boxed{Mx - Ny \neq 0}$$

$$I.F = \frac{1}{Mx - Ny} = \frac{1}{2xy \cos xy}$$

from(1),

$$\frac{(xy \cdot \sin xy + \cos xy)\, y}{2xy \cos xy}\, dx + \frac{(xy \sin xy - \cos xy)\, x}{2xy \cos xy}\, dy = 0$$

$$\left(\frac{xy^2 \sin xy}{2xy \cos xy} + \frac{y \cdot \cos xy}{2xy \cos xy}\right) dx + \left(\frac{x^2 y \sin xy}{2xy \cos xy} - \frac{x \cdot \cos xy}{2xy \cos xy}\right) dy = 0$$

$$\left(\frac{y}{2} \tan xy + \frac{1}{2x}\right) dx + \left(\frac{x}{2} \tan xy - \frac{1}{2y}\right) dy = 0$$

$\rightarrow (2)$

Equⁿ ② is an exact.

where $M = \frac{y}{2}\tan xy + \frac{1}{2x}$ and $N = \frac{x}{2}\tan xy - \frac{1}{2y}$

$$\frac{\partial M}{\partial y} = \frac{1}{2}\left[y\cdot\sec^2 xy(x) + \tan xy(1)\right] + 0$$

$$= \frac{1}{2}\left[xy\sec^2 xy + \tan xy\right]$$

$$= \frac{1}{2}xy\sec^2 xy + \frac{1}{2}\tan xy.$$

and $N = \frac{x}{2}\tan xy - \frac{1}{2y}$

$$\frac{\partial N}{\partial x} = \frac{1}{2}\left[x\cdot\sec^2 xy(y) + \tan xy(1)\right] - 0$$

$$= \frac{1}{2}\left[xy\sec^2 xy + \tan xy\right]$$

$$= \frac{1}{2}xy\sec^2 xy + \frac{1}{2}\tan xy$$

$$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

Clearly equⁿ ② is an exact.

Now the solution of equⁿ ② is $\int M\,dx + \int N\,dy = c$

$$\int\left(\frac{y}{2}\tan xy + \frac{1}{2x}\right)dx + \int\left(\frac{x}{2}\tan xy - \frac{1}{2y}\right)dy = c$$

$$\frac{y}{2}\int \tan xy\,dx + \frac{1}{2}\int\frac{1}{x}dx + \int\frac{x}{2}\tan xy - \frac{1}{2}\int\frac{1}{y}dy = c$$

$$\cancel{\frac{y}{2}}\cdot\frac{\log(\sec xy)}{\cancel{y}} + \frac{1}{2}\log x + 0 - \frac{1}{2}\log y = \log c$$

$$\frac{1}{2}\left[\log(\sec xy) + \log x - \log y\right] = \log c$$

$$\log(\sec xy\cdot x) - \log y = 2\log c$$

$$\cancel{\log}\left(\frac{x\cdot\sec xy}{y}\right) = \cancel{\log}\, c^2$$

$$\frac{x}{y}\cdot\sec xy = c.$$

(2) $(1+xy)\,y\,dx + (1-xy)\,x\,dy = 0$ $\longrightarrow$ ①

Sol:- Equⁿ ① is an exact form of $M\,dx + N\,dy = 0$

where $M = y + xy^2$ and $N = x - x^2y$

$$\frac{\partial M}{\partial y} = 1 + x\cdot 2y = 2xy + 1 \qquad \frac{\partial N}{\partial x} = 1 - y\cdot 2x = 1 - 2xy$$

$$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

clearly equⁿ① is non-exact.

Equⁿ① can be reduced to exact by multiplying Integrating factor.

$$Mx - Ny = x(y + xy^2) - (x - x^2y) \cdot y$$

$$= \cancel{xy} + x^2y^2 - \cancel{xy} + x^2y^2$$

$$= 2x^2y^2 \neq 0$$

$$\boxed{Mx - Ny \neq 0}$$

$$IF = \frac{1}{Mx - Ny} = \frac{1}{2x^2y^2}$$

from①
$$\frac{(y + xy^2)}{2x^2y^2} dx + \frac{(x - x^2y)}{2x^2y^2} dy = 0$$

$$\left(\frac{y}{2x^2y^2} + \frac{xy^2}{2x^2y^2}\right) dx + \left(\frac{x}{2x^2y^2} - \frac{x^2y}{2x^2y^2}\right) dy = 0$$

$$\left(\frac{1}{2x^2y} + \frac{1}{2x}\right) dx + \left(\frac{1}{2xy^2} - \frac{1}{2y}\right) dy = 0 \rightarrow ②$$

Equⁿ② is an exact form of $M dx + N dy = 0$

where $M = \frac{1}{2x^2y} + \frac{1}{2x}$ and $N = \frac{1}{2xy^2} - \frac{1}{2y}$

$$\frac{\partial M}{\partial y} = \frac{1}{2x^2}\left(\frac{-1}{y^2}\right) + 0 \qquad \frac{\partial N}{\partial x} = \frac{1}{2y^2}\left(\frac{-1}{x^2}\right) - 0$$

$$= \frac{-1}{2x^2y^2} \qquad = \frac{-1}{2x^2y^2}$$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

clearly Equⁿ② is an exact.

Now the solution of equⁿ② is $\int M dx + \int N dy = c$

$$\int \left(\frac{1}{2x^2y} + \frac{1}{2x}\right) dx + \int \left(\frac{1}{2xy^2} - \frac{1}{2y}\right) dy = c$$

$$\frac{1}{2y} \int x^{-2} dx + \frac{1}{2} \int \frac{1}{x} dx + \int \frac{1}{2xy^2} dy - \frac{1}{2} \int \frac{1}{y} dy = c$$

$$\frac{1}{2y} \frac{x^{-1}}{-1} + \frac{1}{2} \log x + 0 - \frac{1}{2} \log y = c$$

$$-\frac{1}{2xy} + \frac{1}{2} \log x - \frac{1}{2} \log y = c$$

$$+\frac{1}{2}\left[-\log y + \log x - \frac{1}{xy}\right] = c$$

$$\frac{1}{2}\left[\log\left(\frac{x}{y}\right) - \frac{1}{xy}\right] = c$$

(3) $y(2xy+1)\,dx + x(1+2xy-x^3y^3)\,dy = 0$ $\rightarrow$ ①

Sol:- Equ^n ① is an exact form $Mdx+Ndy=0$.

Where $M = y(2xy+1)$ and $N = x(1+2xy-x^3y^3)$

$= 2xy^2+y$ $\qquad$ $= x+2x^2y-x^4y^3$

$\frac{\partial M}{\partial y} = 2x(2y)+1$ $\qquad$ $\frac{\partial N}{\partial x} = 1+2y(2x)-y^3 4x^3$

$= 4xy+1$ $\qquad$ $= 1+4xy-4x^3y^3$

$$\boxed{\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

Clearly Equ^n ① is non-exact.

Equ^n ① can be converted to exact by multiplying Integrating factor.

$$Mx-Ny = (2xy^2+y)x - (x+2x^2y-x^4y^3)y$$
$$= 2x^2y^2 + \cancel{xy} - \cancel{xy} - 2x^2y^2 + x^4y^4$$
$$= x^4y^4$$

$$I.F = \frac{1}{Mx-Ny} = \frac{1}{x^4y^4}$$

from ①,

$$\frac{y(2xy+1)}{x^4y^4}\,dx + \frac{x(1+2xy-x^3y^3)}{x^4y^4}\,dy = 0$$

$$\left(\frac{2xy^2}{x^4y^4} + \frac{y}{x^4y^4}\right)dx + \left(\frac{x}{x^4y^4} + \frac{2x^2y}{x^4y^4} - \frac{x^4y^3}{x^4y^4}\right)dy = 0$$

$$\left(\frac{2}{x^3y^2} + \frac{1}{x^4y^3}\right)dx + \left(\frac{1}{x^3y^4} + \frac{2}{x^2y^3} - \frac{1}{y}\right)dy = 0 \rightarrow ②$$

Equ^n ① is an exact form of $Mdx+Ndy=0$

Where $M = \frac{2}{x^3y^2} + \frac{1}{x^4y^3}$ and $N = \frac{1}{x^3y^4} + \frac{2}{x^2y^3} - \frac{1}{y}$

$\frac{\partial M}{\partial y} = \frac{2}{x^3}(-2)y^{-3} + \frac{1}{x^4}(-3)y^{-4}$ $\qquad$ $\frac{\partial N}{\partial x} = \frac{1}{y^4}(-3)x^{-4} + \frac{2}{y^3}(-2)x^{-3} + 0$

$= \frac{-4}{x^3y^3} - \frac{3}{x^4y^4}$ $\qquad$ $= \frac{-3}{x^4y^4} - \frac{4}{x^3y^3}$

$$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

clearly Equation is an exact.

Now the solution of equ^n ② is

$$\int M\,dx + \int N\,dy = c.$$

$$\int\left(\frac{2}{x^3y^2}+\frac{1}{x^4y^3}\right)dx+\int\left(\frac{1}{x^3y^4}+\frac{2}{x^2y^3}-\frac{1}{y}\right)dy=c$$

$$\frac{2}{y^2}\int(x^{-3})dx+\frac{1}{y^3}\int x^{-4}dx+\int\frac{1}{x^3y^4}dy+\int\frac{2}{x^2y^3}dy-\int\frac{1}{y}dy=c$$

$$\frac{2}{y^2}\left(\frac{x^{-2}}{-2}\right)+\frac{1}{y^3}\left(\frac{x^{-3}}{-3}\right)+0+0-\log y=c$$

$$\frac{-1}{x^2y^2}-\frac{1}{3x^3y^3}-\log y=c$$

$$\cancel{\frac{-1}{x^2y^2}\left[1+\frac{3}{xy}+\log y\right]=c}$$

$$\frac{-1}{x^2y^2}\left[1+\frac{1}{3xy}\right]-\log y=c.$$

Tuesday 24/09

METHOD – III, IV

(1) $(xy^2-e^{1/x^3})dx-x^2y\,dy=0. \rightarrow (1)$

Sol:- Equ^n (1) is an exact form of $Mdx+Ndy=0$

where $M=xy^2-e^{1/x^3}$ and $N=-x^2y$

$$\frac{\partial M}{\partial y}=x(2y)-0 \qquad \frac{\partial N}{\partial x}=-y(2x)$$

$$=2xy \qquad =-2xy.$$

$$\boxed{\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}}$$

Hence equ^n (1) is non-exact.

This can be reduced to exact by multiplying an integrating factor.

$$\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}=2xy-(-2xy)$$

$$=4xy$$

$$\Rightarrow \frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}=\frac{4xy}{-x^2y}=\frac{-4}{x}$$

Now I.F $e^{\int f(x)dx}$

$$=e^{\int-\frac{4}{x}dx}$$

$$=e^{-4\int\frac{1}{x}dx}$$

$$=e^{-4\log x}$$

$$=e^{\log_e x^{-4}}$$

$$=\frac{1}{x^4}$$

from ①, $\frac{(xy^2 - e^{1/x^3})}{x^4} dx - \frac{x^2 y}{x^4} dy = 0$

$$\frac{xy^2}{x^4} - \frac{e^{1/x^3}}{x^4} dx - \frac{x^2 y}{x^4} dy = 0$$

$$\left(\frac{y^2}{x^3} - x^{-4} e^{1/x^3}\right) dx - \frac{y}{x^2} dy = 0 \rightarrow ②$$

Equⁿ ② is an exact form of $M dx + N dy = 0$

Where $M = \frac{y^2}{x^3} - x^{-4} e^{1/x^3}$ and $N = -\frac{y}{x^2}$

$$\frac{\partial M}{\partial y} = \frac{1}{x^3}(2y) - 0 \qquad \frac{\partial N}{\partial x} = -y(-2)x^{-3}$$

$$= \frac{2y}{x^3} \qquad = \frac{2y}{x^3}$$

$$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

Clearly equⁿ ② is an exact.

Now the solution of equⁿ ② is $\int M dx + \int N dy = c$

$$\int \left(\frac{y^2}{x^3} - x^{-4} \cdot e^{1/x^3}\right) dx + \int \frac{-y}{x^2} dy = c$$

$$y^2 \int x^{-3} dx - \int x^{-4} \cdot e^{x^{-3}} dx + 0 = c$$

$$y^2 \cdot \frac{x^{-2}}{-2} - \int e^t \left(-\frac{1}{3} dt\right) = c \qquad x^{-3} = t$$

$$-3 \cdot x^{-4} dx = dt$$

$$\frac{-2}{x^2 y^2} + \frac{1}{3} \int e^t dt = c \qquad x^{-4} dx = -\frac{1}{3} dt$$

$$\frac{-2}{x^2 y^2} + \frac{1}{3} e^t = c$$

$$\frac{-2}{x^2 y^2} + \frac{1}{3} e^{1/x^3} = c.$$

(2). $(xy^3 + y) dx + 2(x^2y^2 + x + y^4) dy = 0 \rightarrow ①$

Sol: Equⁿ ① is an exact form of $M dx + N dy = 0$

Where $M = xy^3 + y$ and $N = 2x^2y^2 + 2x + 2y^4$

$$\frac{\partial M}{\partial y} = x \cdot 3y^2 + (1) \qquad \frac{\partial N}{\partial x} = 2y^2(2x) + 2(1) + 0$$

$$= 3xy^2 + 1 \qquad = 4xy^2 + 2$$

$$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

Hence equⁿ ① is non exact.

This can be reduced to exact by multiplying an Integrating factor.

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 3xy^2+1-(4xy^2+2)$$
$$= 3xy^2+1-4xy^2-2$$
$$= -xy^2-1$$

$$\Rightarrow \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = \frac{-xy^2-1}{2xy^2+x} = \frac{-(xy^2+1)}{x(2xy^2+1)}$$

$$= \frac{-xy^2-1}{xy^3+y}$$

$$= \frac{-(\cancel{xy^2+1})}{y(\cancel{xy^2+1})}$$

$$= \frac{-1}{y}$$

Now I.F $= e^{-\int g(y)\,dy}$

$$= e^{-\int \frac{-1}{y}\,dy}$$

$$= e^{\log_e y}$$

$$= \underline{\underline{y}}.$$

~~from ①,~~

$$\cancel{\frac{(xy^3+y)}{y}\,dx + 2\frac{(x^2y^2+x+y^4)}{y}\,dy = 0}$$

$$\cancel{\left(\frac{xy^{3}}{y}+\frac{y}{y}\right)dx + 2\left[\frac{x^2y^2}{y}+\frac{x}{y}+\frac{y^4}{y}\right]dy = 0}$$

$$\cancel{(xy^2+1)\,dx + 2\left(x^2y+\frac{x}{y}+y^3\right)dy = 0} \rightarrow ②$$

~~Eqn ② is an exact form of $Mdx+Ndy=0$~~

~~where $M = xy^2+1$ and $N = 2\left[x^2y+\frac{x}{y}+y^3\right]$~~

~~$\frac{\partial M}{\partial y} = x\,2y+0 = 2xy$~~ ~~$\frac{\partial N}{\partial x} = y(2x)+\frac{1}{y}$~~

from ①,

$$y(xy^3+y)\,dx + 2y(x^2y^2+x+y^4)\,dy = 0$$

$$(xy^4+y^2)\,dx + 2(x^2y^3+xy+y^5)\,dy = 0 \rightarrow ②$$

Eqn ② is an exact form of $Mdx+Ndy=0$

where $M = xy^4+y^2$ and $N = 2[x^2y^3+xy+y^5]$

$$\frac{\partial M}{\partial y} = x\cdot 4y^3+2y = 4xy^3+2y$$

$$\frac{\partial N}{\partial x} = 2[y^3(2x)+y(1)+0] = 4xy^3+2y$$

$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$ clearly Equn ② is an Exact.

Now the soln of Equn ② is $\int M dx + \int N dy = c$

$$\int (xy^4 + y^2) dx + \int 2(x^2y^3 + xy + y^5) dy = c$$

$$y^4 \int x dx + y^2 \int (1) dx + 2\int x^2y^3 dy + 2\int xy dy + 2\int y^5 dy = C$$

$$y^4 \frac{x^2}{2} + y^2.(x) + 0 + 0 + \cancel{2} \cdot \frac{y^6}{\cancel{6}_3} = C.$$

$$\frac{1}{2} x^2 y^4 + x y^2 + \frac{y^6}{3} = C.$$

$$\frac{3x^2y^4 + 6xy^2 + 2y^6}{6} = C \Rightarrow 3x^2y^4 + 6xy^2 + 2y^6 = 6C$$

$$\Rightarrow \boxed{3x^2y^4 + 6xy^2 + 2y^6 = C}$$

(7) $\left(y + \frac{y^3}{3} + \frac{x^2}{2}\right) dx + \frac{1}{4}(x + xy^2) dy = 0 \longrightarrow ①$

Sol:- Equn ① is an exact form of $M dx + N dy = 0$

where $M = y + \frac{y^3}{3} + \frac{x^2}{2}$ and $N = \frac{1}{4}(x + xy^2)$

$$\frac{\partial M}{\partial y} = 1 + \frac{1}{\cancel{3}}\cancel{3}y^2 + 0 = 1 + y^2$$

$$\frac{\partial N}{\partial x} = \frac{1}{4}(1 + y^2(1)) = \frac{1}{4}(1 + y^2)$$

$$\boxed{\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

Hence equn ① is non-exact.

This can be reduced to exact by multiplying Integrating factor.

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 1 + y^2 - \frac{1}{4}(1 + y^2)$$
$$= 1 + y^2(1 - 1/4)$$

$$\Rightarrow \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{\cancel{(1+y^2)}(1 - 1/4)}{1/4 x \cancel{(1+y^2)}} = \frac{4(1 - 1/4)}{x}$$
$$= \frac{4 - 1}{x}$$
$$= \frac{3}{x}$$

Now I.F $e^{\int f(x) dx}$

$$= e^{\int \frac{3}{x} dx}$$
$$= e^{3\int \frac{1}{x} dx}$$
$$= e^{3 \log x}$$
$$= e^{\log_e x^3}$$
$$= x^3$$

from ①, $\left(y.x^3 + \frac{x^3y^3}{3} + \frac{x^5}{2}\right) dx + \frac{1}{4}(x^4 + x^4y^2) dx = 0$

$\longrightarrow ②$

Equ^n ② is an exact form of $Mdx + Ndy = 0$.

where $M = yx^3 + \frac{x^3y^3}{3} + \frac{x^5}{2}$ and $N = \frac{1}{4}(x^4 + x^4y^2)$

$\frac{\partial M}{\partial y} = x^3(1) + \frac{x^3}{\cancel{3}} \cdot \cancel{3}y^2 + 0$

$= x^3(1+y^2)$

$\frac{\partial N}{\partial x} = \frac{1}{4}(4x^3 + 4x^3y^2)$

$= \frac{\cancel{4}x^3}{\cancel{4}}(1+y^2)$

$= x^3(1+y^2)$

$$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

clearly Equ^n ② is an exact.

Now the solution of equ^n ② is $\int M dx + \int N dy = c$

$\int \left(x^3y + \frac{x^3y^3}{3} + \frac{x^5}{2}\right) dx + \int \frac{1}{4}(x^4 + x^4y^2)\, dy = c$

$y\int x^3 dx + \frac{y^3}{3}\int x^3 dx + \frac{1}{2}\int x^5 dx + 0 = c$

$y\frac{x^4}{4} + \frac{y^3}{3}\frac{x^4}{4} + \frac{1}{2}\frac{x^6}{6} = c$

$\frac{x^4y}{4} + \frac{x^4y^3}{12} + \frac{x^6}{12} = c$

$\frac{3x^4y + x^4y^3 + x^6}{12} = c$

$3x^4y + x^4y^3 + x^6 = 12c$

$3x^4y + x^4y^3 + x^6 = c$.

(8) $(x\sec^2 y - x^2\cos y)\, dy = (\tan y - 3x^4)\, dx$

Sol: $(\tan y - 3x^4)dx - (x\sec^2 y - x^2\cos y)\, dy = 0 \rightarrow ①$

Equ^n ① is an exact form of $Mdx + Ndy = 0$

where $M = \tan y - 3x^4$ and $N = +x^2\cos y - x\sec^2 y$

$\frac{\partial M}{\partial y} = \sec^2 y - 0$

$= (\sec^2 y)$

$\frac{\partial N}{\partial x} = \cos y(2x) - \sec^2 y(1)$

$= 2x\cos y - \sec^2 y$

$$\boxed{\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

clearly equ^n ① is non-exact.

This can be reduced to exact by multiplying Integrating factor.

$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \sec^2 y - 2x\cos y + \sec^2 y$

$= 2\sec^2 y - 2x\cos y$

$$\Rightarrow \frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N} = \frac{2\sec^2 y - 2x\cos y}{-(x\sec^2 y - x^2\cos y)}$$

$$= \frac{2(\sec^2 y - x\cos y)}{-x(\sec^2 y - x\cos y)}$$

$$= \frac{-2}{x}$$

Now I.F $e^{\int f(x)dx}$

$$= e^{\int \frac{-2}{x}dx}$$

$$= e^{-2\log x}$$

$$= \frac{1}{x^2}.$$

from ①, $\frac{(\tan y - 3x^4)}{x^2}dx - \frac{(x\sec^2 y - x^2\cos y)}{x^2}dy = 0$

$$\left(\frac{\tan y}{x^2} - \frac{3x^{4}}{x^2}\right)dx - \left(\frac{x\sec^2 y}{x^2} - \frac{x^2\cos y}{x^2}\right)dy = 0$$

$$\left(\frac{\tan y}{x^2} - 3x^2\right)dx - \left(\frac{\sec^2 y}{x} - \cos y\right)dy = 0 \rightarrow ②$$

Equⁿ ② is an exact form of $Mdx + Ndy = 0$

Where $M = \frac{\tan y}{x^2} - 3x^2$ and $N = \cos y - \frac{\sec^2 y}{x}$

$$\frac{\partial M}{\partial y} = \frac{1}{x^2}\sec^2 y - 0 = \frac{\sec^2 y}{x^2}$$

$$\frac{\partial N}{\partial x} = 0 - \sec^2 y \cdot \left(\frac{-1}{x^2}\right) = \frac{\sec^2 y}{x^2}$$

$$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

Clearly equⁿ ② is an exact.

Now the solution of equⁿ ② is $\int M dx + \int N dy = c$

$$\int\left(\frac{\tan y}{x^2} - 3x^2\right)dx + \int\left(\cos y - \frac{\sec^2 y}{x}\right)dy = c$$

$$\tan y\int x^{-2}dx - 3\int x^2 dx + \int \cos y\, dy - \int \frac{\sec^2 y}{x}dy = c$$

$$\tan y\left(\frac{x^{-1}}{-1}\right) - \cancel{3}\frac{x^3}{\cancel{3}} + \sin y - 0 = c$$

$$-\frac{\tan y}{x} - x^3 + \sin y = c$$

$$\frac{1}{x}\tan y + x^3 - \sin y = c.$$

(9) $\left(xy\, e^{x/y}+y^2\right)dx - x^2 e^{x/y}\, dy = 0 \rightarrow (1)$ (I - Method)

Sol: Equn (1) is an exact form of $Mdx+Ndy=0$

where $M = xy e^{x/y}+y^2$

$$\frac{\partial M}{\partial y} = x\left[y\cdot e^{x/y}\left(\frac{-x}{y^2}\right)+e^{x/y}(1)\right]+2y$$

$$= x\left[e^{x/y}\frac{-x}{y}+e^{x/y}\right]+2y$$

$$= x\cdot e^{x/y}\left(1-\frac{x}{y}\right)+2y$$

and $N = -x^2 e^{x/y}$

$$\frac{\partial N}{\partial x} = -\left(x^2\cdot e^{x/y}\cdot\frac{1}{y}+e^{x/y}\cdot 2x\right)$$

$$= -\, x\cdot e^{x/y}\left[\frac{x}{y}+2\right]$$

$$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

Hence equn (1) is non-exact.

→ This can be reduced to exact by multiplying Integrating factor.

→ clearly equn (1) is a homogeneous ~~of~~ degree '2'.

$$Mx+Ny = \left(xye^{x/y}+y^2\right)x + \left(-x^2e^{x/y}\right)y$$

$$= \cancel{x^2y\cdot e^{x/y}} + xy^2 - \cancel{x^2ye^{x/y}}$$

$$= xy^2 \neq 0$$

$$\boxed{Mx+Ny \neq 0}$$

Now $\quad I.F = \dfrac{1}{Mx+Ny} = \dfrac{1}{xy^2}$

from (1),

$$\frac{\left(xye^{x/y}+y^2\right)}{xy^2}dx - \frac{x^2e^{x/y}}{xy^2}dy = 0$$

$$\left(\frac{\cancel{xy}e^{x/y}}{\cancel{xy}y}+\frac{\cancel{y^2}}{x\cancel{y^2}}\right)dx - \frac{x\cancel{x}e^{x/y}}{\cancel{x}y^2}dy = 0$$

$$\left(\frac{e^{x/y}}{y}+\frac{1}{x}\right)dx - \frac{xe^{x/y}}{y^2}\cdot dy = 0 \rightarrow (2)$$

Equn (2) is an exact form of $Mdx+Ndy=0$

where $M = \dfrac{e^{x/y}}{y}+\dfrac{1}{x}$.

$$\frac{\partial M}{\partial y} = \frac{\cancel{y}\cdot e^{x/y}\cdot x\left(\frac{-1}{y\cancel{^2}}\right) - e^{x/y}(1)}{y^2} + 0$$

$$= \frac{-x/y\, e^{x/y} - e^{x/y}}{y^2}$$

$$= \frac{-x/y . e^{x/y}}{y^2} - \frac{e^{x/y}}{y^2}$$

$$= \frac{-x e^{x/y}}{y^3} - \frac{e^{x/y}}{y^2}$$

$$= -\frac{e^{x/y}}{y^2}\left(\frac{x}{y}+1\right)$$

and $N = \frac{-x . e^{x/y}}{y^2}$

$$\frac{\partial N}{\partial x} = \frac{-1}{y^2}\left[x . e^{x/y} \frac{1}{y}(1) + e^{x/y}.(1)\right]$$

$$= \frac{-1}{y^2}\left[\frac{x}{y} e^{x/y} + e^{x/y}\right]$$

$$= -\frac{e^{x/y}}{y^2}\left(\frac{x}{y}+1\right)$$

$$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

clearly equn ① is an exact.

Now the solution of equn ① is $\int M dx + \int N dy = C$

$$\int\left(\frac{e^{x/y}}{y} + \frac{1}{x}\right)dx + \int -\frac{x e^{x/y}}{y^2}\, dy = c$$

$$\frac{1}{y}\int e^{x/y} dx + \int \frac{1}{x} dx - 0 = C$$

$$\frac{1}{y} . e^{x/y} \frac{1}{y}(1) + \log x = c$$

$$\frac{e^{x/y}}{y^2} + \log x = C$$

(10) $(3xy - 2ay^2)\, dx + (x^2 - 2axy)\, dy = 0 \rightarrow ①$

Sol: Equn ① is an exact form of $M dx + N dy = 0$

where $M = 3xy - 2ay^2$ and $N = x^2 - 2axy$

$$\frac{\partial M}{\partial y} = 3x(1) - 2a(2y) \qquad \frac{\partial N}{\partial x} = 2x - 2ay(1)$$

$$= 3x - 4ay \qquad = 2x - 2ay$$

$$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

Hence equn ① is non-exact.

This can be reduced to exact by multiplying Integrating factor.

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 3x - 4ay - 2x + 2ay$$

$$= x - 2ay.$$

$$\Rightarrow \quad \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{x - 2ay}{x^2 - 2axy}$$

$$= \frac{\cancel{x - 2ay}}{x(\cancel{x - 2ay})}$$

$$= \frac{1}{x}.$$

Now $I.F = e^{\int f(x)\,dx} = e^{\int \frac{1}{x} dx}$

$$= e^{\log_e x}$$

$$= x.$$

from ①,

$$\frac{(3xy - 2ay^2)}{x} dx + \left(\frac{x^2 - 2axy}{x}\right) dy = 0$$

$$\left(\frac{3\cancel{x}y}{\cancel{x}} - \frac{2ay^2}{x}\right) dx + \left(\frac{x^{\cancel{2}}}{\cancel{x}} - \frac{2a\cancel{x}y}{\cancel{x}}\right) dy = 0$$

$$\left(3y - \frac{2ay^2}{x}\right) dx + (x - 2ay)\, dy = 0 \rightarrow ②$$

Equ② is an exact form of $Mdx + Ndy = 0$.

from ①,

$$(3xy - 2ay^2)\, x \, dx + (x^2 - 2axy)\, x \, dy = 0$$

$$(3x^2y - 2axy^2)\, dx + (x^3 - 2ax^2y)\, dy = 0 \longrightarrow ②$$

Equ② is an exact form of $Mdx + Ndy = 0$

where $M = 3x^2y - 2axy^2$ and $N = x^3 - 2ax^2y$

$$\frac{\partial M}{\partial y} = 3x^2(1) - 2ax(2y) \qquad \frac{\partial N}{\partial x} = 3x^2 - 2ay(2x)$$

$$= 3x^2 - 4axy \qquad = 3x^2 - 4axy$$

$$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

Clearly equ② is an exact.

Now the solution of equ② is $\int M dx + \int N dy = c$

$$\int (3x^2y - 2axy^2)\, dx + \int (x^3 - 2ax^2y)\, dy = c$$

$$3y \int x^2 dx - 2ay^2 \int (x)\, dx + 0 = c$$

$$\cancel{3}y \frac{x^3}{\cancel{3}} - \cancel{2}ay^2 \frac{x^2}{\cancel{2}} = c$$

$$x^3y - ax^2y^2 = c$$

$$x^2y(x - ay) = c$$

(11) $(x^4e^x - 2mxy^2)\,dx + 2mx^2y\,dy = 0$. $\rightarrow$ (1)

Sol: Equn (1) is an exact form of $Mdx + Ndy = 0$

where $M = x^4e^x - 2mxy^2$ and $N = 2mx^2y$

$$\frac{\partial M}{\partial y} = 0 - 2mx(2y) = -4mxy$$

$$\frac{\partial N}{\partial x} = 2my(2x) = 4mxy$$

$$\boxed{\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

Hence Equn (1) is non-exact.
This can be reduced to exact by multiplying Integrating factor.

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = -4mxy - 4mxy = -8mxy.$$

$$\Rightarrow \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{-\cancel{8}^{4}\cancel{mxy}}{\cancel{2m}x^{\cancel{2}}\cancel{y}} = \frac{-4}{x}.$$

Now
$$I.F = e^{\int f(x)\,dx} = e^{-4\int \frac{1}{x}dx} = e^{-4\log x} = e^{\log_e(x)^{-4}} = x^{-4} = \frac{1}{x^4}.$$

from (1),

$$\left(\frac{x^4e^x - 2mxy^2}{x^4}\right)dx + \left(\frac{2mx^2y}{x^4}\right)dy = 0$$

$$\left(\frac{\cancel{x^4}e^x}{\cancel{x^4}} - \frac{2m\cancel{x}y^2}{x^{\cancel{4}3}}\right)dx + \left(\frac{2m\cancel{x^2}y}{x^{\cancel{4}2}}\right)dy = 0$$

$$\left(e^x - \frac{2my^2}{x^3}\right)dx + \left(\frac{2my}{x^2}\right)dy = 0 \rightarrow (2)$$

Equn (2) is an exact form of $Mdx + Ndy = 0$.

where $M = e^x - \frac{2my^2}{x^3}$ and $N = \frac{2my}{x^2}$

$$\frac{\partial M}{\partial y} = 0 - \frac{2m}{x^3}(2y)$$
$$= \frac{-4my}{x^3}$$

$$\frac{\partial N}{\partial x} = 2my \cdot (-2)\, x^{-3}$$
$$= \frac{-4my}{x^3}.$$

$$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

Clearly equ^n (2) is an exact.

Now the solution of equ^n (1) is $\int M dx + \int N dy = c$

$$\int \left(e^x - \frac{2my^2}{x^3}\right) dx + \int \frac{2my}{x^2}\, dy = c$$

$$\int e^x . dx - 2my^2 \int x^{-3} dx + 0 = c$$

$$e^x - \cancel{2}my^2\left(\frac{x^{-2}}{-\cancel{2}}\right) = c$$

$$e^x + \frac{my^2}{x^2} = c.$$

(12) $y \cdot (2x^2y + e^x) \cdot dx = (e^x + y^3)\, dy.$

Sol:- $y.(2x^2y + e^x)\, dx = (e^x + y^3)\, dy$

$$(2x^2y^2 + y.e^x)\, dx - (e^x + y^3)\, dy = 0 \rightarrow (1)$$

Equ^n (1) is an exact form of $Mdx + Ndy = 0$

where $M = 2x^2y^2 + y.e^x$ and $N = -(e^x + y^3)$

$$\frac{\partial M}{\partial y} = 2x^2 \cdot (2y) + e^x (1)$$
$$= 4x^2y + e^x$$

$$\frac{\partial N}{\partial x} = -(e^x + 0)$$
$$= -e^x.$$

$$\boxed{\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

Hence equ^n (1) is non-exact.

This can be reduced to exact by multiplying Integrating factor.

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 4x^2y + e^x - (-e^x)$$
$$= 4x^2y + e^x + e^x$$
$$= 4x^2y + 2e^x.$$

$$\Rightarrow \frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{M} = \frac{4x^2y+2e^x}{2x^2y^2+y\cdot e^x}$$

$$= \frac{2\,\cancel{(2x^2y+e^x)}}{y\,\cancel{(2x^2y+e^x)}}$$

$$= \frac{2}{y}.$$

Now $\text{I.F} = e^{-\int g(y)\,dy} = e^{-\int \frac{2}{y}dy}$

$$= e^{-2\cdot \log y}$$

$$= e^{\log_e (y)^{-2}}$$

$$= \frac{1}{y^2}.$$

from ①,

$$\frac{(2x^2y^2+y\cdot e^x)}{y^2}\,dx - \frac{(e^x+y^3)}{y^2}\,dy = 0$$

$$\left(\frac{2x^2\cancel{y^2}}{\cancel{y^2}}+\frac{\cancel{y}\cdot e^x}{y^{\cancel{2}}}\right)dx - \left(\frac{e^x}{y^2}+\frac{y^{\cancel{3}}}{\cancel{y^2}}\right)dy = 0$$

$$\left(2x^2+\frac{e^x}{y}\right)dx - \left(\frac{e^x}{y^2}+y\right)dy = 0 \rightarrow ②$$

Equⁿ ② is an exact form of $Mdx+Ndy=0$

where $M = 2x^2+\frac{e^x}{y}$ and $N = -\left(\frac{e^x}{y^2}+y\right)$

$$\frac{\partial M}{\partial y} = 0+e^x\cdot\frac{-1}{y^2} = \frac{-e^x}{y^2} \qquad \frac{\partial N}{\partial x} = -\left(\frac{1}{y^2}e^x+0\right) = \frac{-e^x}{y^2}$$

$$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

clearly Equⁿ ② is an exact.

Now the solⁿ of Equⁿ ② is $\int M dx + \int N dy = c$

$$\int\left(2x^2+\frac{e^x}{y}\right)dx + \int -\left(\frac{e^x}{y^2}+y\right)dy = c$$

$$2\int x^2\,dx + \frac{1}{y}\int e^x\,dx - \int \frac{e^x}{y^2}\,dy - \int y\,dy = c$$

$$2\frac{x^3}{3}+\frac{1}{y}e^x - 0 - \frac{y^2}{2} = c$$

$$\frac{2x^3}{3}+\frac{e^x}{y}-\frac{y^2}{2} = c.$$

(15) $(3x^2y^4 + 2xy)\,dx + (2x^3y^3 - x^2)\,dy = 0 \rightarrow (1)$

Sol: Equn (1) is an exact form of $Mdx + Ndy = 0$

Where $M = 3x^2y^4 + 2xy$ and $N = 2x^3y^3 - x^2$

$$\frac{\partial M}{\partial y} = 3x^2 \cdot 4y^3 + 2x \qquad \frac{\partial N}{\partial x} = 2y^3\,3x^2 - 2x$$

$$= 12x^2y^3 + 2x \qquad = 6x^2y^3 - 2x.$$

$$\boxed{\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

Hence Equn (1) is non-exact.

This can be reduced to exact by multiplying an Integrating factor.

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 12x^2y^3 + 2x - 6x^2y^3 + 2x$$

$$= 6x^2y^3 + 4x.$$

$$= 2(3x^2y^3 + 2x)$$

$$\Rightarrow \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = \frac{2(3x^2y^3 + 2x)}{3x^2y^4 + 2xy} = \frac{2\cancel{(3x^2y^3 + 2x)}}{y\cancel{(3x^2y^3 + 2x)}}$$

$$= \frac{2}{y}$$

$$I.F \quad e^{-\int g(y)\,dy} = e^{-\int \frac{2}{y}dy}$$

$$= e^{-2\cdot\log y}$$

$$= e^{\log(y)^{-2}}$$

$$= \frac{1}{y^2}.$$

from (1),

$$\left(\frac{3x^2y^4 + 2xy}{y^2}\right)dx + \left(\frac{2x^3y^3 - x^2}{y^2}\right)dy = 0$$

$$\left(\frac{3x^2y^4}{y^2} + \frac{2xy}{y^2}\right)dx + \left(\frac{2x^3y^3}{y^2} - \frac{x^2}{y^2}\right)dy = 0$$

$$\left(3x^2y^2 + \frac{2x}{y}\right)dx + \left(2x^3y - \frac{x^2}{y^2}\right)dy = 0 \rightarrow (2)$$

Equn (2) is an exact form of $Mdx + Ndy = 0$

Where $M = 3x^2y^2 + \frac{2x}{y}$ and $N = 2x^3y - \frac{x^2}{y^2}$

$$\frac{\partial M}{\partial y} = 3x^2(2y) + 2x\left(\frac{-1}{y^2}\right) \qquad \frac{\partial N}{\partial x} = 2y(3x^2) - \frac{1}{y^2}(2x)$$

$$= 6x^2y - \frac{2x}{y^2} \qquad = 6x^2y - \frac{2x}{y^2}$$

$$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

Clearly Equ^n ② is an exact.

Now the sol^n of Equ^n ② is $\int M dx + \int N dy = C$

$$\int \left(3x^2y^2 + \frac{2x}{y}\right) dx + \int \left(2x^3y - \frac{x^2}{y^2}\right) dy = C$$

$$3y^2 \int x^2 dx + \frac{2}{y} \int x dx + 0 = C$$

$$\cancel{3}y^2 \frac{x^3}{\cancel{3}} + \frac{\cancel{2}}{y} \cdot \frac{x^2}{\cancel{2}} = C$$

$$x^3y^2 + x^2 \frac{1}{y} = C.$$

(16) $y \log y \, dx + (x - \log y) \, dy = 0 \rightarrow ①$

Sol:- Equ^n ① is an exact form of $Mdx + Ndy = 0$

where $M = y \log y$ and $N = x - \log y$

$$\frac{\partial M}{\partial y} = \cancel{y} \cdot \frac{1}{\cancel{y}} + \log y (1) \qquad \frac{\partial N}{\partial x} = 1 - 0$$

$$= 1 + \log y \qquad = 1.$$

$$\boxed{\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

Hence Equ^n ① is non-exact.

This can be reduced to exact by multiplying an Integrating factor.

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \cancel{1} + \log y - \cancel{1}$$

$$= \log y.$$

$$\Rightarrow \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = \frac{\log y}{y \log y} = \frac{1}{y}.$$

Now I.F $e^{-\int g(y) dy}$

$$= e^{-\int \frac{1}{y} dy}$$

$$= e^{-\log y}$$

$$= e^{\log (y)^{-1}}$$

$$= y^{-1}$$

$$= \frac{1}{y}.$$

from ①, $\frac{\cancel{y}\log y}{\cancel{y}} dx + \left(\frac{x-\log y}{y}\right) dy = 0.$

$$\log y \cdot dx + \left(\frac{x}{y} - \frac{\log y}{y}\right) dy = 0 \rightarrow ②$$

Equⁿ ② is an exact form of $Mdx + Ndy = 0$

where $M = \log y$. and $N = \frac{x}{y} - \frac{\log y}{y}$

$\frac{\partial M}{\partial y} = \frac{1}{y}$. $\qquad \frac{\partial N}{\partial x} = \frac{1}{y}(1) - 0$.

$= \frac{1}{y}$.

$$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

clearly equⁿ ② is an exact.

Now the solⁿ of equⁿ ② is $\int M dx + \int N dy = c$.

$$\int \log y \cdot dx + \int\left(\frac{x}{y} - \frac{\log y}{y}\right) dy = c$$

$$\log y \int(1) dx + \int \frac{x}{y} dy - \int \frac{\log y}{y} dy = c$$

$$\log y \cdot (x) + 0 - \int t \cdot dt = c. \qquad \log y = t$$

$$x \cdot \log y - \frac{t^2}{2} = c \qquad \frac{1}{y} dy = dt.$$

$$x \cdot \log y - \frac{(\log y)^2}{2} = c.$$

③ $(x^2+y^2+x)dx + xy\, dy = 0. \rightarrow ①$

Sol:- Equⁿ ① is an exact form of $Mdx + Ndy = 0$

where $M = x^2+y^2+x$ and $N = xy$

$\frac{\partial M}{\partial y} = 0 + 2y + 0$ $\qquad \frac{\partial N}{\partial x} = y \cdot (1)$

$= 2y$ $\qquad = y$

$$\boxed{\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

Hence equⁿ ① is non-exact.

This can be reduced to exact by multiplying an Integrating factor.

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 2y - y = y.$$

$$\Rightarrow \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{\cancel{y}}{x\cancel{y}} = \frac{1}{x}.$$

Now I.F $e^{\int f(x)dx} = e^{\int \frac{1}{x} dx}$

$= e^{\log_e x}$

$= \underline{x}$

from ①,

$\frac{x^2+y^2+x}{x} . dx + \frac{xy}{x} dy = 0$

$\left(\frac{x^4}{x} + \frac{y^2}{x} + \frac{x}{x}\right) dx + y.dy = 0$

$\left(x + \frac{y^2}{x} + 1\right) dx + y.dy = 0 \rightarrow ②$

Equⁿ ② is an exact form of $Mdx + Ndy = 0$

$2 \underline{| 2, 4, 3}$
$\quad 1, 2, 3$

Where $M = x + \frac{y^2}{x} + 1$

$\frac{\partial M}{\partial y} = 0 + \frac{1}{x}(2y) + 0$

$= \frac{2y}{x}$

from ①,

$x(x^2+y^2+x) dx + x(xy) dy = 0$

$(x^3 + xy^2 + x^2) dx + x^2y \, dy = 0 \rightarrow ②$

Equⁿ ② is an exact form of $Mdx + Ndy = 0$

Where $M = x^3 + xy^2 + x^2$ and $N = x^2y$

$\frac{\partial M}{\partial y} = 0 + x(2y) + 0$ $\qquad \frac{\partial N}{\partial x} = y(2x)$

$= 2xy$ $\qquad = 2xy$

$$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

clearly Equⁿ ② is an exact.

Now the solⁿ of Equⁿ ② is $\int M dx + \int N dy = c$

$\int (x^3 + xy^2 + x^2) dx + \int (x^2y) dy = c$

$\int x^3 dx + y^2 \int x \, dx + \int x^2 dx + 0 = c$

$\frac{x^4}{4} + y^2 \cdot \frac{x^2}{2} + \frac{x^3}{3} = c$

$\frac{3x^4 + 6x^2y^2 + 4x^3}{12} = c$

$3x^4 + 6x^2y^2 + 4x^3 = 12c$

$3x^4 + 6x^2y^2 + 4x^3 = c.$

(4) $(x^2+y^2+1)\,dx - 2xy\,dy = 0. \rightarrow ①$

Sol:- Equⁿ ① is an exact form of $Mdx+Ndy=0$.

Where $M = x^2+y^2+1$ and $N = -2xy$

$\frac{\partial M}{\partial y} = 0+2y+0$ $\quad$ $\frac{\partial N}{\partial x} = -2y(1)$

$= 2y$ $\quad$ $= -2y$

$$\boxed{\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

Hence Equⁿ① is a non-exact.

This can be reduced to exact by multiplying an Integrating factor.

$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 2y-(-2y) = 2y+2y = \underline{4y}$

$\Rightarrow \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{4y}{-2xy} = \frac{-2}{x}$

Now I.F $e^{\int f(x)dx} = e^{\int \frac{-2}{x}dx}$

$= e^{-2\cdot \log x}$

$= e^{\log_e (x)^{-2}}$

$= \frac{1}{x^2}$

from①, $\frac{x^2+y^2+1}{x^2}\cdot dx - \frac{2xy}{x^2}\cdot dy = 0$

$\left(\frac{x^2}{x^2}+\frac{y^2}{x^2}+\frac{1}{x^2}\right)dx - \frac{2y}{x}dy = 0$

$\left(1+\frac{y^2}{x^2}+\frac{1}{x^2}\right)dx - \frac{2y}{x}\cdot dy = 0 \rightarrow ②$

Equⁿ② is an exact form of $Mdx+Ndy=0$

Where $M = 1+\frac{y^2}{x^2}+\frac{1}{x^2}$ and $N = \frac{-2y}{x}$

$\frac{\partial M}{\partial y} = 0+\frac{1}{x^2}(2y)+0$ $\quad$ $\frac{\partial N}{\partial x} = -2y\left(\frac{-1}{x^2}\right)$

$= \frac{2y}{x^2}$ $\quad$ $= \frac{2y}{x^2}$

$$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

Clearly Equⁿ② is an Exact.

Now the solu^n of Equ^n ② is $\int M dx + \int N dy = c$

$$\int\left(1+\frac{y^2}{x^2}+\frac{1}{x^2}\right)dx + \int \frac{-2y}{x}\cdot dy = c$$

$$\int (1)\,dx + y^2\int x^{-2}dx + \int x^{-2}dx - 0 = C.$$

$$x + y^2\cdot\frac{x^{-1}}{-1} + \frac{x^{-1}}{-1} = C$$

$$x - \frac{y^2}{x} - \frac{1}{x} = C.$$

$$\frac{x^2-y^2-1}{x} = C$$

$$\frac{1}{x}(x^2-y^2-1) = C.$$

⑤ $(x^2+y^2+2x)\,dx + 2y\,dy = 0. \rightarrow ①$

Sol:- Equ^n ① is an Exact form of $Mdx+Ndy=0$

where $M = x^2+y^2+2x$ and $N = 2y$

$$\frac{\partial M}{\partial y} = 0+2y+0 \qquad \frac{\partial N}{\partial x} = 0$$

$$= 2y \qquad\qquad = 0$$

$$\boxed{\because \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

Hence Equ^n ① is non-Exact.

This can be reduced to Exact by multiplying an Integrating factor.

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 2y - 0 = 2y.$$

$$\Rightarrow \frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N} = \frac{\cancel{2y}}{\cancel{2y}} = \underline{1}$$

Now, I.F $e^{\int f(x)dx} = e^{\int (1)dx}$

$$= \underline{\underline{e^x}}.$$

from ①, $e^x(x^2+y^2+2x)\,dx + 2y\cdot e^x\,dy = 0$

$$(e^x x^2 + e^x y^2 + 2x\cdot e^x)\,dx + 2ye^x\cdot dy = 0 \rightarrow ②$$

Equ^n ② is an Exact form of $Mdx+Ndy=0$

where $M = e^x x^2 + e^x\cdot y^2 + 2x\cdot e^x$

$$\frac{\partial M}{\partial y} = \cancel{e^x(2x)+2e^x} \; 0 + e^x\cdot(2y) + 0$$

$$= e^x\cdot 2y$$

and $N = 2y \cdot e^x$

$$\frac{\partial N}{\partial x} = 2y(e^x)$$
$$= 2y \cdot e^x$$

$$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

clearly Equn ② is an Exact.

Now the solun of equn ② is $\int M dx + \int N dy = c$

$$\int (e^x \cdot x^2 + e^x y^2 + 2x \cdot e^x)\, dx + \int 2y e^x \cdot dy = c.$$

$$\int e^x \cdot x^2 dx + y^2 \int e^x \cdot dx + 2\int x \cdot e^x dx + 0 = c.$$

D	I
$+x^2$	e^x
$-2x$	e^x
$+2$	e^x
-0	e^x.

$$x^2 e^x - 2x \cdot e^x + 2 \cdot e^x + y^2 \cdot e^x + 2e^x \cdot (x-1) = c$$

$$x^2 e^x - \cancel{2x \cdot e^x} + \cancel{2e^x} + y^2 \cdot e^x + \cancel{2x \cdot e^x} - \cancel{2e^x} = c$$

$$x^2 \cdot e^x + y^2 \cdot e^x = c$$

$$e^x (x^2 + y^2) = c.$$

(6) $(y^4 + 2y)\, dx + (xy^3 + 2y^4 - 4x)\, dy = 0 \longrightarrow ①$

Sol:- Equn ① is an exact ~~form~~ of $Mdx + Ndy = 0$

where $M = y^4 + 2y$

$$\frac{\partial M}{\partial y} = 4y^3 + 2$$
$$= 2(2y^3 + 1)$$

and $N = xy^3 + 2y^4 - 4x$

$$\frac{\partial N}{\partial x} = y^3(1) + 0 - 4$$
$$= y^3 - 4$$

$$\boxed{\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

Hence equn ① is non-Exact.

This can be reduced to exact by multiplying an Integrating factor.

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 4y^3 + 2 - (y^3 - 4)$$
$$= 4y^3 + 2 - y^3 + 4$$
$$= 3y^3 + 6.$$

$$\Rightarrow \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = \frac{3y^3 + 6}{y^4 + 2y} = \frac{3\cancel{(y^3+2)}}{y\cancel{(y^3+2)}} = \frac{3}{y}.$$

Now I.F $e^{-\int g(y)\,dy} = e^{-\int \frac{3}{y}\,dy}$

$= e^{-3\log y}$.

$= e^{\log_e (y)^{-3}}$

$= \frac{1}{y^3}$.

from ①, $\left(\frac{y^4+2y}{y^3}\right) dx + \left(\frac{xy^3+2y^4-4x}{y^3}\right) dy = 0$

$\left(\frac{y^4}{y^3} + \frac{2y}{y^3}\right) dx + \left(\frac{xy^3}{y^3} + \frac{2y^4}{y^3} - \frac{4x}{y^3}\right) dy = 0$

$\left(y + \frac{2}{y^2}\right) dx + \left(x + 2y - \frac{4x}{y^3}\right) dy = 0 \rightarrow ②$

Equn ② is an exact form of $Mdx + Ndy = 0$

where $M = y + \frac{2}{y^2}$ and $N = x + 2y - \frac{4x}{y^3}$

$\frac{\partial M}{\partial y} = 1 + 2 \cdot (-2) y^{-3}$ $\qquad$ $\frac{\partial N}{\partial x} = 1 + 0 - \frac{4}{y^3}(1)$

$= 1 - \frac{4}{y^3}$ $\qquad$ $= 1 - \frac{4}{y^3}$

$$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

clearly equn ② is an exact.

Now the solun of equn ② is $\int M dx + \int N dy = C$

$\int \left(y + \frac{2}{y^2}\right) dx + \int \left(x + 2y - \frac{4x}{y^3}\right) dy = C$

$y\int (1) dx + \frac{2}{y^2} \int (1) dx + \int x \cdot dy + 2\int y\, dy - \int \frac{4x}{y^3} \cdot dy = C$

$y(x) + \frac{2}{y^2} \cdot (x) + 0 + \cancel{2} \cdot \left(\frac{y^2}{\cancel{2}}\right) - 0 = C$

$xy + \frac{2x}{y^2} + y^2 = C.$

(13) $\cdot\, y\,dx - x\,dy + \log x \cdot dx = 0.$

Sol:- $(y + \log x)\, dx - x\, dy = 0 \rightarrow ①$

Equn ① is an exact form of $Mdx + Ndy = 0$

where $M = y + \log x$ and $N = -x$

$\frac{\partial M}{\partial y} = 1 + 0$ $\qquad$ $\frac{\partial N}{\partial x} = -(1)$

$= 1$ $\qquad$ $= -1$

$$\boxed{\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

Hence Equ^n ① is non-Exact.

This ~~eq~~ can be reduced to exact by multiplying an Integrating factor.

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 1-(-1) = 1+1 = \underline{2}$$

$$\Rightarrow \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{2}{-x} = \frac{-2}{x}$$

Now I.F $e^{\int f(x)dx} = e^{\int -\frac{2}{x}dx}$

$$= e^{-2\int \frac{1}{x}dx}$$

$$= e^{-2\log x}$$

$$= e^{\log_e (x)^{-2}}$$

$$= \frac{1}{x^2}$$

from①,

$$\left(\frac{y+\log x}{x^2}\right)dx - \left(\frac{x}{x^2}\right)dy = 0$$

$$\left(\frac{y}{x^2} + \frac{\log x}{x^2}\right)dx - \left(\frac{1}{x}\right)dy = 0 \quad \to ②$$

Equ^n ② is an exact form of $Mdx + Ndy = 0$

where $M = \frac{y}{x^2} + \frac{\log x}{x^2}$ and $N = \frac{-1}{x}$

$$\frac{\partial M}{\partial y} = \frac{1}{x^2}(1) + 0 \qquad \frac{\partial N}{\partial x} = (-1)\frac{-1}{x^2}$$

$$= \frac{1}{x^2} \qquad\qquad = \frac{1}{x^2}$$

$$\boxed{\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

clearly Equ^n ② is an Exact.

Now the solu^n of ~~eqn~~ Equ^{n}② is $\int M dx + \int N dy = c$

$$\int\left(\frac{y}{x^2} + \frac{\log x}{x^2}\right)dx + \int\left(\frac{-1}{x}\right)dy = c.$$

$$y\int x^{-2}dx + \int \log x \cdot \frac{1}{x^2}\cdot dx - 0 = c.$$

$$y\cdot\left(\frac{x^{-1}}{-1}\right) + \int \log x \cdot x^{-2}\cdot dx = c.$$

Integration By parts.

D	I
$\log x$	x^{-2}
$\frac{1}{x}$	$\frac{x^{-1}}{-1} = \frac{-1}{x}$.

$$\frac{-1}{xy} + \left[\log x \cdot \left(\frac{-1}{x}\right) - \int \left(\frac{1}{x}\right)\left(\frac{-1}{x}\right) dx\right] = C$$

$$\frac{-1}{xy} - \frac{\log x}{x} + \int x^{-2}\, dx = C.$$

$$\frac{-1}{xy} - \frac{\log x}{x} + \frac{x^{-1}}{-1} = C.$$

$$\frac{-1}{xy} - \frac{\log x}{x} - \frac{1}{x} = C$$

$$-\frac{1}{x}\left[\frac{1}{y} + \log x + 1\right] = C.$$

$$\frac{-1}{x}\left[\frac{1 + y\cdot\log x + y}{y}\right] = C$$

$$\frac{-1}{xy}\left[1 + y + y\cdot\log x\right] = C.$$

(14) $(2x\log x - xy)\,dy + 2y\,dx = 0.$

$2y\cdot dx + (2x\log x - xy)\,dy = 0 \longrightarrow (1)$

Sol:- Equn (1) is an Exact form of $Mdx + Ndy = 0$

where $M = 2x\cdot\log x - xy$ and $N = 2y$

$\frac{\partial M}{\partial y} = 0 - x\cdot(1)$ $\qquad$ $\frac{\partial N}{\partial x} = 0.$

$= -x$ $\qquad\qquad$ $= 0$

$$\boxed{\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

Hence equn (1) is non-Exact.

This can be reduced to Exact by multiplying an Integrating factor.

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = -x - 0 = -x.$$

$$\Rightarrow \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{-x}{2y}$$

where $M = 2y$ and $N = 2x\log x - xy$

$\frac{\partial M}{\partial y} = 2\cdot(1)$ $\qquad$ $\frac{\partial N}{\partial x} = 2\cdot\left[x\cdot\frac{1}{x} + \log x(1)\right] - y(1)$

$= 2$ $\qquad\qquad$ $= 2(1 + \log x) - y$

$$\boxed{\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

Hence equn (1) is non-Exact.

This can be reduced to Exact by multiplying an Integrating facto

$$\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x} = 2-[2(1+\log x)-y]$$

$$= \cancel{2}-\cancel{2}-2\log x+y$$

$$= y-2\log x$$

$$\Rightarrow \frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N} = \frac{y-2\log x}{2x\log x-xy} = \frac{-(2\log x-y)}{x(2\log x-y)} = \frac{-1}{x}.$$

Now. I.F $e^{\int f(x)dx} = e^{\int -\frac{1}{x}dx}$

$$= e^{-\log x}$$

$$= e^{\log_e (x)^{-1}}$$

$$= \frac{1}{x}$$

from ①,

$$\left(\frac{2y}{x}\right)dx+\left(\frac{2x\log x-xy}{x}\right)dy = 0$$

$$\left(\frac{2y}{x}\right)dx+\left(\frac{2\cancel{x}\log x}{\cancel{x}}-\frac{\cancel{x}y}{\cancel{x}}\right)dy = 0$$

$$\left(\frac{2y}{x}\right)dx+(2\log x-y)\,dy = 0 \rightarrow ②$$

Equⁿ ② is an Exact form of $Mdx+Ndy=0$

where $M=\frac{2y}{x}$ and $N=2\log x-y$

$\frac{\partial M}{\partial y}=\frac{2}{x}(1)$ $\qquad$ $\frac{\partial N}{\partial x}=2\cdot\left(\frac{1}{x}\right)-0$

$=\frac{2}{x}$ $\qquad$ $=\frac{2}{x}$

$$\boxed{\therefore \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}}$$

Clearly Equⁿ ② is an Exact.

Now the solutⁿ of Equⁿ ② is $\int Mdx+\int Ndy = C$

$$\int\left(\frac{2y}{x}\right)dx+\int(2\log x-y)\,dy = C$$

$$2y\int\frac{1}{x}dx+\int 2\log x\cdot dy-\int y\,dy = C$$

$$2y\cdot\log x+0-\frac{y^2}{2} = C$$

$$2y\log x-\frac{y^2}{2} = C$$

$$\frac{4y\cdot\log x-y^2}{2} = C$$

$$4y\log x-y^2 = 2C$$

$$4y\log x-y^2 = C.$$

Thursday
26/09/19

Inspection Method.

(3) $y(2xy+e^x)\,dx = e^x.dy$

Sol: $(2xy^2+ye^x)\,dx = e^x dy$

$2xy^2\,dx + ye^x\,dx = e^x dy$

$2xy^2\,dx + ye^x dx - e^x dy = 0$

$\frac{2xy^2}{y^2}dx + \frac{ye^x dx - e^x dy}{y^2} = 0$

$2x\,dx + d\left(\frac{e^x}{y}\right) = 0$

$2\int x\,dx + \int d\left(\frac{e^x}{y}\right) = C$

$\cancel{2}\left(\frac{x^2}{\cancel{2}}\right) + \frac{e^x}{y} = C$

$x^2 + \frac{e^x}{y} = C.$

(4) $(y\log y - 2xy)\,dx + (x+y)\,dy = 0$

Sol:- $y\log y\,dx - 2xy\,dx + x\,dy + y\,dy = 0.$

$y\log y\,dx + y\,dy + x.dy - 2xy\,dx = 0.$

$y\log y \overset{dx}{+} x.dy - 2xy\,dx + y\,dy = 0$

$\frac{\cancel{y}\log y\,dx}{\cancel{y}} + \frac{x}{y}dy - \frac{2x\cancel{y}}{\cancel{y}}dx + \frac{\cancel{y}}{\cancel{y}}dy = 0$

$\log y.dx + \frac{1}{y}.x\,dy - 2x\,dx + dy = 0$

$d.(\log y.x) - 2x\,dx + dy = 0$

$\int d(\log y.x) - 2\int x\,dx + \int (1)\,dy = C$

$\log y.x - \cancel{2}.\frac{x^2}{\cancel{2}} + y. = C$

$x.\log y - x^2 + y = C.$

(7) $x\,dy - y\,dx = (4x^2+y^2)\,dy$

Sol:- $x\,dy - y\,dx = 4x^2\,\cancel{dy} + y^2\,dy$

$x\,dy - y\,dx - 4x^2\,dy + y^2\,dy = 0$

(9) $(x+y)^2 \cdot \left(x \cdot \frac{dy}{dx} + y\right) = xy\left(1 + \frac{dy}{dx}\right)$

Sol:- $(x+y)^2 \left(\frac{x\,dy + y\,dx}{\cancel{dx}}\right) = xy\left(\frac{dx + dy}{\cancel{dx}}\right)$

$$(x+y)^2 \cdot (x\,dy + y\,dx) = xy\,(dx + dy)$$

$$\frac{x\,dy + y\,dx}{xy} = \frac{dx + dy}{(x+y)^2}$$

$$d\left(\log(xy)\right) = -\left(\frac{-1}{(x+y)^2}\right)(dx+dy)$$

$$d\left(\log(x\cdot y)\right) = -\,d\left(\frac{1}{x+y}\right)$$

$$\int d\left(\log(x.y)\right) + \int d\left(\frac{1}{x+y}\right) = C$$

$$\log(x.y) + \frac{1}{x+y} = C.$$

(7) $x\,dy - y\,dx = (4x^2+y^2)\,dy$

Sol:- $x\,dy - y\,dx = ((2x)^2 + y^2)\,dy$

$$\frac{x\,dy - y\,dx}{(2x)^2 + y^2} = dy$$

$$\frac{1}{2}\,d\left(\tan^{-1}\left(\frac{y}{2x}\right)\right) = dy$$

$$\frac{1}{2}\,d\left(\tan^{-1}\left(\frac{y}{2x}\right)\right) - dy = 0$$

$$\frac{1}{2}\int d\left(\tan^{-1}\left(\frac{y}{2x}\right)\right) - \int (1)\,dy = C$$

$$\frac{1}{2}\tan^{-1}\left(\frac{y}{2x}\right) - y = C$$

$\because\ d\left(\tan^{-1}\left(\frac{y}{2x}\right)\right) = \frac{1}{1+\left(\frac{y}{2x}\right)^2}\left(\frac{2x.dy - y.2dx}{(2x)^2}\right)$

$$= \frac{2\,(x\,dy - y\,dx)}{\left(1 + \frac{y^2}{(2x)^2}\right)(2x)^2}$$

$$= \frac{2\,(x\,dy - y\,dx)}{\frac{(2x)^2 + y^2}{\cancel{(2x)^2}}\ \cancel{(2x)^2}}$$

$$\frac{1}{2}\,d\left(\tan^{-1}\left(\frac{y}{2x}\right)\right) = \frac{x\,dy - y\,dx}{(2x)^2 + y^2}$$

(10) $x\,dy - y\,dx = x\sqrt{x^2 - y^2}\,dx$

Sol:- $x\,dy - y\,dx = x\sqrt{x^2\left(1 - \frac{y^2}{x^2}\right)}\,dx$

$$x\,dy - y\,dx = x^2\sqrt{1 - \left(\frac{y}{x}\right)^2}\,dx$$

$$\frac{x\,dy - y\,dx}{x^2\sqrt{1 - \left(\frac{y}{x}\right)^2}} = dx$$

$$d\left(\sin^{-1}\left(\frac{y}{x}\right)\right) - dx = 0.$$

$$\int d\left(\sin^{-1}\left(\frac{y}{x}\right)\right) - \int (1)\,dx = C$$

$$\sin^{-1}\left(\frac{y}{x}\right) - x = C.$$

(5) $x\,dy - y\,dx = xy^2\,dx$

Sol:- $-(y\,dx - x\,dy) = xy^2\,dx$

$$\frac{y\,dx - x\,dy}{y^2} = -x\,dx.$$

$$d\left(\frac{x}{y}\right) + x\,dx = 0$$

$$\int d\left(\frac{x}{y}\right) + \int x\,dx = c$$

$$\frac{x}{y} + \frac{x^2}{2} = c.$$

(6) $x\,dy = (x^2y^2 - y)\,dx.$

Sol:- $x\,dy = x^2y^2\,dx - y\,dx.$

$$x\,dy + y\,dx = x^2y^2\,dx$$

$$x\,dy + y\,dx = (xy)^2\,dx$$

$$\frac{x\,dy + y\,dx}{(xy)^2} = dx$$

$$-d\left(\frac{1}{xy}\right) = dx$$

$$dx + d\left(\frac{1}{xy}\right) = 0$$

$$\int (1)\,dx + \int d\left(\frac{1}{xy}\right) = C$$

$$x + \frac{1}{xy} = C$$

(8) $(y + y^2\cos x)\,dx - (x - y^3)\,dy = 0$

Sol:- $y\,dx + y^2\cos x\,dx - x\,dy + y^3\,dy = 0.$

$$y\,dx - x\,dy + y^3\,dy = -y^2\cos x\,dx.$$

$$\frac{y\,dx - x\,dy}{y^2} + \frac{y^3\,dy}{y^2} = -\cos x\,dx$$

$$d\left(\frac{x}{y}\right) + y\,dy + \cos x\,dx = 0.$$

$$\int d\left(\frac{x}{y}\right) + \int y\,dy + \int \cos x\,dx = C$$

$$\frac{x}{y} + \frac{y^2}{2} + \sin x = C.$$

(11) $x\,dx + y\,dy - a^2\,d\left(\tan^{-1}\left(\frac{y}{x}\right)\right) = 0.$

Sol:- $x\,dx + y\,dy - a^2\,d\left(\tan^{-1}\left(\frac{y}{x}\right)\right) = 0$

$$\int x\,dx + \int y\,dy - a^2 \int d\left(\tan^{-1}\left(\frac{y}{x}\right)\right) = C$$

$$\frac{x^2}{2} + \frac{y^2}{2} - a^2 \tan^{-1}\left(\frac{y}{x}\right) = C.$$

APPLICATIONS OF FIRST ORDER DIFFERENTIAL EQUATIONS

Monday 30/09/2019

(5) $y^2 = \frac{x^3}{a-x}$ (orthogonal trajectory)

Sol: $y^2 = \frac{x^3}{a-x} \rightarrow ①$

$y^2(a-x) = x^3 \rightarrow ①$

Diff. Eqn① w.r.t. 'x'

$$\frac{d}{dx}(y^2(a-x)) = \frac{d}{dx}(x^3)$$

$$y^2(0-1)+(a-x)2y\frac{dy}{dx} = 3x^2$$

$$-y^2+(a-x)2y\frac{dy}{dx} = 3x^2$$

$$2y\cdot\frac{dy}{dx}(a-x)-y^2 = 3x^2$$

$$2y\cdot\frac{dy}{dx}(a-x) = 3x^2+y^2$$

$$2yy'(a-x) = 3x^2+y^2$$

$$\boxed{a-x = \frac{3x^2+y^2}{2yy'}}$$

$$a = \frac{3x^2+y^2}{2yy'}+x$$

from①,

$$y^2\left(\frac{3x^2+y^2}{2yy'}\right) = x^3$$

$$y^{\cancel{2}}(3x^2+y^2) = 2\cancel{y}y'x^3$$

$$3x^2y+y^3 = 2\cdot\frac{dy}{dx}x^3 \rightarrow ②$$

Replace $\frac{dy}{dx}$ by $-\frac{dx}{dy}$ by $\frac{dy}{dx}$.

$$3x^2y+y^3 = 2\frac{-dx}{dy}x^3$$

$$3x^2y+y^3 = -2\cdot\frac{dx}{dy}x^3$$

$$-2x^3\frac{dx}{dy} = 3x^2y+y^3 \rightarrow ③$$

$$-2x^3\cdot dx = (3x^2y+y^3)dy$$

$$(3x^2y+y^3)dy+2x^3\cdot dx = 0.$$

$$\frac{dx}{dy} = \frac{-(3x^2y + y^3)}{2x^3}$$

$$\frac{dx}{dy} = \frac{-3x^2y}{2x^3} - \frac{y^3}{2x^3}$$

$$\frac{dx}{dy} = -\frac{3y}{2x} - \frac{y^3}{2x^3}$$

$$\frac{dx}{dy} + \left(\frac{3}{2x}\right)y = \frac{-y^3}{2} \cdot x^{-3}. \quad \text{(Bernoulli's)}$$

put $y = vx \Rightarrow \boxed{v = \frac{y}{x}}$

$\underline{dy} = x\,dv.$

$$\frac{\pm dx}{x\,dv} = \frac{-(3x^2(vx) + (vx)^3)}{2x^3}$$

$$\frac{dx}{x\,dv} = \frac{-(3x^3v + v^3x^3)}{2x^3}$$

$$\frac{1}{x}\cdot\frac{dx}{dv} = -\frac{\cancel{x^3}(3v + v^3)}{2\cancel{x^3}}$$

$$\frac{-2}{x}\cdot dx = (3v + v^3)dv$$

$$-2\int \frac{1}{x}\,dx = 3\int v\,dv + \int v^3\,dv$$

$$-2\log x = 3\left(\frac{v^2}{2}\right) + \left(\frac{v^4}{4}\right) + c$$

$$-2\log x = \frac{3}{2}(v^2) + \frac{1}{4}(v)^4 + c$$

$$-2\log x = \frac{3}{2}\left(\frac{y^2}{x^2}\right) + \frac{1}{4}\left(\frac{y}{x}\right)^4 + c.$$

(6) $y = \dfrac{x^3 - a^3}{3x}$

Sol:- $3xy = x^3 - a^3 \rightarrow ①$

Diff. equⁿ ① w.r.to 'x'.

$$3\left[x\cdot\frac{dy}{dx} + y\cdot\frac{dx}{dx}\right] = 3x^2 - 0.$$

$$\cancel{3}\left[x\cdot\frac{dy}{dx} + y\cdot\ \right] = \cancel{3}x^2$$

$$x\cdot\frac{dy}{dx} + y = x^2 \rightarrow ②$$

Replace $-\dfrac{dx}{dy}$ by $\dfrac{dy}{dx}$

$$x\cdot\left(-\frac{dx}{dy}\right)+y = x^2$$

$$y-\frac{dx}{dy}(x) = x^2$$

$$y-\frac{dx}{dy}\cdot x = x^2$$

$$x\frac{dx}{dy} = y-x^2$$

$$x\,dx = (y-x^2)\,dy$$

$$x\,dx-(y-x^2)\,dy = 0.$$

$M = x$ and $N = -(y-x^2)$

$\frac{\partial M}{\partial y} = 0$ $\qquad\frac{\partial N}{\partial x} = -(0-2x) = 2x.$

$$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$

Hence eqn ② is non exact.

This can be reduced to exact by multiplying an integrating factor.

$\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x} = 0-2x = -2x$ $\Rightarrow \frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{M} = \frac{-2\cancel{x}}{\cancel{x}} = -2$

$$\text{I.F } e^{-\int g(y)\,dy} = e^{+2\int(1)\,dy} = e^{+2y}.$$

$$e^{2y}\left[x\cdot dx-(y-x^2)\,dy\right] = 0$$

$$e^{2y}\int x\cdot dx-\int e^{2y}y\,dy+\int\frac{x^2}{e^{2y}}\,dy = 0.$$

$$e^{2y}\left(\frac{x^2}{2}\right)-\left[\frac{e^{2y}}{2}\cdot y-\frac{e^{2y}}{4}\right]+0 = 0$$

$$\frac{1}{2}\cdot x^2e^{2y}-\left[\frac{e^{2y}}{2}\cdot y-\frac{1}{4}\cdot e^{2y}\right] = 0.$$

$$\frac{1}{2}e^{2y}\left(x^2-y+\frac{1}{2}\right) = 0.$$

$$\frac{1}{2}e^{2y}\left(x^2-y+1/2\right) = 0.$$

(7) $y^2 = ax^3$ → ①

Sol:- diff. eqn ① w.r.to 'x'

$$2y \cdot \frac{dy}{dx} = a \cdot 3x^2$$

$$2y \cdot \frac{dy}{dx} = 3ax^2$$

$$2y \cdot \frac{dy}{dx} = 3ax^2$$

$$a = \frac{2y}{3x^2} \cdot \frac{dy}{dx}$$

$$\boxed{a = \frac{2yy'}{3x^2}}$$

from ①,

$$y^{\cancel{2}} = \left(\frac{2\cancel{y}y'}{3\cancel{x^2}}\right)x^{\cancel{3}}$$

$$y = \frac{2y'x}{3}$$

$$3y = 2xy'$$

$$3y = 2x\frac{dy}{dx}$$

Replace $\frac{dy}{dx} = -\frac{dx}{dy}$

$$3y = 2x\left(-\frac{dx}{dy}\right)$$

$$3y + 2x \cdot \frac{dx}{dy} = 0$$

$$2x \cdot dx = 3y\,dy$$

$$\frac{\cancel{2}x^2}{\cancel{2}} = 3\frac{y^2}{2} + c$$

$$x^2 = \frac{y^2}{2} + c.$$

(8) $y = c(\sec x + \tan x)$ → ①

Sol:- differentiate with respect to 'x'

$$\frac{dy}{dx} = c(\sec x \cdot \tan x + \sec^2 x)$$

$$y' = c\left(\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} + \frac{1}{\cos^2 x}\right)$$

$$y' = c\left(\frac{\sin x + 1}{\cos^2 x}\right)$$

$$y' = c\left(\frac{\sin x + 1}{1-\sin^2 x}\right)$$

$$y' = c\left(\frac{\cancel{\sin x + 1}}{(1+\sin x)(1-\sin x)}\right)$$

$$y' = \frac{c}{1-\sin x}.$$

$$\boxed{c = (1-\sin x)\,y'.}$$

from ①,

$$y = (1-\sin x)\,y'(\sec x + \tan x)$$

$$y = y'(1-\sin x)\left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right)$$

$$y = y'(1-\sin x)\left(\frac{1+\sin x}{\cos x}\right)$$

$$y = y'\left(\frac{1-\sin^2 x}{\cos x}\right)$$

$$y = y'\left(\frac{\cos^2 x}{\cancel{\cos x}}\right)$$

$$y = y'\cos x.$$

$$y = \frac{dy}{dx}\cdot\cos x. \implies \text{Replace } \frac{dy}{dx} = -\frac{dx}{dy}$$

$$\frac{1}{\cos x}\cdot dx = \frac{1}{y}\,dy \qquad\qquad y = \frac{-dx}{dy}\cdot\cos x$$

$$\int \sec x\,dx = \int \frac{1}{y}\,dy \qquad\qquad y\,dy = -\cos x\,dx$$

$$\log(\sec x + \tan x) = \log y + \log c \qquad \int y\,dy = -\int \cos x\,dx$$

$$\log(\sec x + \tan x) = \log(c\cdot y) \qquad \frac{y^2}{2} = -\sin x + c$$

$$\boxed{cy = \sec x + \tan x} \qquad \boxed{\frac{y^2}{2} + \sin x = c}$$

③ Find the particular no. of orthogonal trajectories $x^2 + cy^2 = 1$ passing through the point $(2,1)$.

Soln $\quad x^2 + cy^2 = 1 \quad \rightarrow ①$

diff w. r. to 'x'

$$2x + c\cdot 2y\,\frac{dy}{dx} = 0.$$

$$\cancel{2}x = -\cancel{2}cy\,\frac{dy}{dx}$$

$$x = -cy\,\frac{dy}{dx}$$

$$x = -c\,y \cdot y'$$

$$\boxed{c = \frac{-x}{y y'}}$$

from ①,

$$x^2 + \left(\frac{-x}{y y'}\right) y^{\cancel{2}} = 1$$

$$x^2 - \frac{xy}{y'} = 1$$

$$x^2 = 1 + \frac{xy}{y'}$$

$$x^2 - 1 = xy \cdot \frac{1}{y'}$$

Replace $\frac{dy}{dx} = -\frac{dx}{dy}$

$$x^2 - 1 = xy \cdot \frac{1}{-\frac{dx}{dy}}$$

$$x^2 - 1 = xy\left(-\frac{dy}{dx}\right)$$

$$\frac{x^2-1}{x} \cdot dx = -y \cdot dy$$

$$\left(\frac{x^{\cancel{2}}}{\cancel{x}} - \frac{1}{x}\right) dx = -y \cdot dy$$

$$\int x \cdot dx - \int \frac{1}{x} \cdot dx = -\int y\, dy$$

$$\frac{x^2}{2} - \log x = \frac{y^2}{2} + c$$

$$\frac{x^2}{2} - \log x = -\frac{y^2}{2} + c$$

$$\frac{x^2}{2} + \frac{y^2}{2} - \log x = c.$$

$$\frac{x^2}{2} + \frac{y^2}{2} = \log x + c$$

Given that the curve passes through the point $(2,1)$

$$\frac{(2)^{\cancel{2}}}{\cancel{2}} + \frac{(1)^2}{2} = \log 2 + c$$

$$2 + \frac{1}{2} = \log 2 + c$$

$$\frac{5}{2} = 0.301 + c$$

$$2.5 = 0.301 + C$$

$$C = 2.5 - 0.301$$

$$\boxed{C = 2.199}$$

Approximately $C = 2.2$

(9) $x^2+y^2+2gx+c=0$ where 'g' is the parameter.

Sol:- $x^2+y^2+2gx+c=0 \rightarrow (1)$

diff. w. r. to 'x'.

$$2x+2y\cdot\frac{dy}{dx}+2g+0=0$$

$$x+y\frac{dy}{dx}+g=0$$

$$x+y\cdot y'+g=0.$$

$$g=-(x+yy')$$

from (1),

$$x^2+y^2+2(-(x+yy'))x+c=0 \rightarrow (2).$$

$$x^2+y^2-2x^2-2xyy'+c=0$$

$$-x^2+y^2-2xyy'+c=0$$

$$c=x^2-y^2+2xyy'$$

from (2)

$$\cancel{x^2+y^2+2\overset{-2(x+yy')}{g}x+x^2-y^2+2xyy'=0}.$$

$$\cancel{2x+2yy'+2(2x)}.$$

$$x^2+\cancel{y^2}-2x-2yy'+x^2-\cancel{y^2}+2xyy'=0$$

$$2x^2-2x-2yy'+2xyy'=0$$

$$2x^2-2x-2y\frac{dy}{dx}+2xy\frac{dy}{dx}=0$$

Replace $\frac{dy}{dx} = -\frac{dx}{dy}$

$$2x^2-2x+2y\frac{dx}{dy}-2xy\frac{dx}{dy}=0.$$

$$2x^2-2x+(2y-2xy)\frac{dx}{dy}=0.$$

$$\cancel{2}(x^2-x)=-\cancel{2}(y-xy)\frac{dx}{dy}$$

$$x^2-x=-y(1-x)\frac{dx}{dy}.$$

$$x(\cancel{x-1})=y(\cancel{x-1})\frac{dx}{dy}$$

$$\frac{1}{y}\cdot dy = \frac{1}{x}\,dx$$

$$\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$$

$$\log y = \log x + \log c$$

$$\cancel{\log} y = \cancel{\log}(c.x)$$

$$\boxed{y = c.x}$$

(10) $y^2 = 4ax$

Sol: $y^2 - 4ax = 0 \rightarrow ①$

diff. w.r.to 'x'

$$2y\,y' - 4a = 0.$$

$$\cancel{4}^{2} a = \cancel{2} y . y'$$

$$a = \frac{yy'}{2}$$

from ①, $y^2 - \cancel{4}x.\left(\frac{yy'}{\cancel{2}}\right) = 0$

$$y^2 - 2xy.y' = 0.$$

$$y^2 - 2xy\frac{dy}{dx} = 0$$

Replace $\frac{dy}{dx} = \frac{-dx}{dy}.$

$$y^2 + 2xy.\frac{dx}{dy} = 0.$$

$$y\cancel{^2} = -2x\cancel{y}.\frac{dx}{dy}$$

$$y\,.dy = -2x.dx.$$

$$\int y\,dy = -2\int x.dx$$

$$\frac{y^2}{2} = -\cancel{(2)}\frac{x^2}{\cancel{2}} + c.$$

$$\frac{x^2}{1} + \frac{y^2}{2} = c.$$

(11) $xy = c.$

Sol: $xy - c = 0 \rightarrow ①$

diff. w.r. to 'x'

$$\left[x.\frac{dy}{dx} + y(1)\right] - 0 = 0.$$

$$x\,y' + y = 0$$

Replace $\frac{dy}{dx} = \frac{-dx}{dy}$

$$x\cdot\left(-\frac{dx}{dy}\right) + y = 0.$$

$$\cancel{-}x\cdot dx = \cancel{-}y\cdot dy.$$

$$\int x\cdot dx = \int y\,dy$$

$$\frac{x^2}{2} = \frac{y^2}{2} + c.$$

$$\frac{x^2}{2} - \frac{y^2}{2} = c.$$

(3) $\cdot e^x + e^{-y} = c.$

Sol:- $e^x + e^{-y} - c = 0 \rightarrow ①$

diff w. r. to 'x'.

$$e^x + e^{-y}\left(-\frac{dy}{dx}\right) - 0 = 0.$$

$$e^x - e^{-y}\cdot\frac{dy}{dx} = 0$$

Replace $\frac{dy}{dx} = \frac{-dx}{dy}$

$$e^x + e^{-y}\,\frac{dx}{dy} = 0.$$

$$e^x = -e^{-y}\cdot\frac{dx}{dy}.$$

$$\frac{1}{e^{-y}}\,dy = \frac{-1}{e^x}\cdot dx$$

$$e^{-y}\,dy = -e^{-x}\,dx$$

$$\int e^{-y}\,dy = -\int e^{-x}\,dx$$

$$e^{-y}(-1) = -e^{-x}\cdot(-1) + c$$

$$-e^{-y} = e^{-x} + c.$$

$$e^{-x} + e^{-y} + c = 0.$$

(4) $x^2 + y^2 = c^2.$

Sol:- $x^2 + y^2 - c^2 = 0 \rightarrow ①$

differentiate w. r. to 'x'

$$2x + 2y\,\frac{dy}{dx} - 0 = 0$$

$$x + y\cdot\frac{dy}{dx} = 0$$

Replace $\frac{dy}{dx} = -\frac{dx}{dy}$

$$x - y \cdot \frac{dx}{dy} = 0$$

$$x = y \frac{dx}{dy}$$

$$\frac{1}{y} dy = \frac{1}{x} dx$$

$$\int \frac{1}{y} dy = \int \frac{1}{x} dx$$

$$\log y = \log x + \log c$$

$$\log y = \log (c \cdot x)$$

$$\boxed{y = cx}$$

Tuesday 01/10/2019

Polar Form

(1). $r = a(1+\cos\theta)$

Sol:- $r = a + a\cos\theta \rightarrow ①$

diff. w. r. to 'θ'.

$$\frac{dr}{d\theta} = 0 + a\cdot(-\sin\theta)$$

$$\frac{dr}{d\theta} = -a\sin\theta \Rightarrow a = \frac{-1}{\sin\theta} \cdot \frac{dr}{d\theta}$$

Replace $\frac{dr}{d\theta} = -r^2 \frac{d\theta}{dr}$

from①,

$$r = \frac{-1}{\sin\theta} \frac{dr}{d\theta} (1+\cos\theta)$$

$$r = \frac{-1}{\sin\theta} \frac{dr}{d\theta} - \cot\theta \frac{dr}{d\theta}$$

Replace $\frac{dr}{d\theta} = -r^2 \frac{d\theta}{dr}$

$$r = -\operatorname{cosec}\theta \left(-r^2 \frac{d\theta}{dr}\right) - \cot\theta \left(-r^2 \frac{d\theta}{dr}\right)$$

$$\cancel{r} = r^{\cancel{2}}\left[\operatorname{cosec}\theta \cdot \frac{d\theta}{dr} + \cot\theta \cdot \frac{d\theta}{dr}\right]$$

$$\frac{1}{r} = (\operatorname{cosec}\theta + \cot\theta) \frac{d\theta}{dr}$$

$$\frac{1}{r} dr = (\operatorname{cosec}\theta + \cot\theta) d\theta$$

$$\int \frac{1}{r} dr = \int \operatorname{cosec}\theta d\theta + \int \cot\theta \cdot d\theta.$$

$$\log r = \log(\operatorname{cosec}\theta - \cot\theta) + \log(\sin\theta) + \log c.$$

$$\log r = \log\left[(\operatorname{cosec}\theta - \cot\theta)(\sin\theta) \cdot c\right]$$

$r = (\mathrm{cosec}\,\theta \cdot \sin\theta - \cot\theta \cdot \sin\theta)\, c$

$r = \left(\frac{1}{\sin\theta} \cdot \sin\theta - \frac{\cos\theta}{\sin\theta} \sin\theta\right) c$

$r = (1 - \cos\theta)\, c.$

(2) $r^n \sin n\theta = a^n$

Sol:- $\log(r^n \cdot \sin n\theta) = \log a^n$

$\log r^n + \log \sin n\theta = n \cdot \log a$

$n \cdot \log r + \log \sin n\theta = n \cdot \log a$

diff. w. r. to 'θ'

$n \cdot \frac{1}{r} \frac{dr}{d\theta} + \frac{1}{\sin n\theta} (\cos n\theta)\, n = 0.$

$\frac{\cancel{n}}{r} \frac{dr}{d\theta} = -\cancel{n} \cdot \cot n\theta$

$\frac{1}{r} \frac{dr}{d\theta} = -\cot n\theta \cdot \cancel{d\theta}$

Replace $\frac{dr}{d\theta} = -r^2 \frac{d\theta}{dr}$

$\frac{1}{\cancel{r}} (\cancel{-} r^{\cancel{2}}) \frac{d\theta}{dr} = \cancel{-} \cot n\theta.$

$r \cdot \frac{d\theta}{dr} = \cot n\theta.$

$\frac{1}{\cot n\theta} d\theta = \frac{1}{r} dr$

$\int \tan n\theta \, d\theta = \int \frac{1}{r} dr$

$\frac{\log(\sec n\theta)}{n} = \log r + \log c$

$\frac{1}{n} \cdot \log(\sec n\theta) = \log(r \cdot c)$

$\cancel{\log} (\sec n\theta)^{1/n} = \cancel{\log} (c \cdot r).$

$(\sec n\theta)^{1/n} = c \cdot r$

$\sec n\theta = (cr)^n$

$\sec n\theta = c \cdot r^n.$

(3). $r^2 = a^2(\cos 2\theta)$

Sol:- $\log r^2 = \log(a^2 \cos 2\theta)$

$2\log r = \log a^2 + \log \cos 2\theta$
$(2\log a)$

diff. w. r. to 'θ'

$2 \cdot \frac{1}{r} \cdot \frac{dr}{d\theta} = 0 + \frac{1}{\cos 2\theta}(-\sin 2\theta) 2.$

$\cancel{2} \cdot \frac{1}{r} \frac{dr}{d\theta} = -(\tan 2\theta) \cancel{2}$

Replace $\frac{dr}{d\theta} = -r^2 \frac{d\theta}{dr}$

$\frac{1}{r}\left(\cancel{-}r^{\cancel{2}} \frac{d\theta}{dr}\right) = \cancel{-}\tan 2\theta$

$\frac{1}{\tan 2\theta} \cdot d\theta = \frac{1}{r} \cdot dr$

$\int \cot 2\theta \cdot d\theta = \int \frac{1}{r} dr$

$\frac{\log(\sin 2\theta)}{2} = \log r + \log c$

$\frac{1}{2} \log(\sin 2\theta) = \log(c \cdot r)$

$\cancel{\log}(\sin 2\theta)^{1/2} = \cancel{\log}(c \cdot r)$

$\sin 2\theta = (c r)^2$

$\sin 2\theta = c \cdot r^2.$

(4) $r^n = a \sin n\theta.$

Sol:- $\log r^n = \log(a \sin n\theta)$

$n \cdot \log r = \log a + \log \sin n\theta.$

diff. w. r. to 'θ'.

$n \cdot \frac{1}{r} \cdot \frac{dr}{d\theta} = 0 + \frac{1}{\sin n\theta}(\cos n\theta) n$

$\cancel{n} \cdot \frac{1}{r} \frac{dr}{d\theta} = \cot n\theta \cdot \cancel{n}$

Replace $\frac{dr}{d\theta} = -r^2 \frac{d\theta}{dr}$

$\frac{1}{r}\left(-r^{\cancel{2}} \frac{d\theta}{dr}\right) = \cot n\theta.$

$-\frac{1}{\cot n\theta} d\theta = \frac{1}{r} dr$

$-\int \tan n\theta \, d\theta = \int \frac{1}{r} dr$

$$-\frac{\log(\sec n\theta)}{n} = \log r + \log c$$

$$-\frac{1}{n}\log(\sec n\theta) = \log(c.r)$$

$$\log(\sec n\theta)^{-1/n} = \log(c.r)$$

$$\sec n\theta = (cr)^{-n}$$

$$\sec n\theta = c^{-n}.r^{-n}$$

$$\sec n\theta = \frac{1}{c^n.r^n}$$

$$c.\sec n\theta = \frac{1}{r^n}.$$

(5) $r = \frac{2a}{1+\cos\theta}$

sol: $r(1+\cos\theta) = 2a$

$(r + r.\cos\theta)$

diff. w.r. to 'θ'

$$\frac{dr}{d\theta} + \left[r.(-\sin\theta) + \cos\theta\frac{dr}{d\theta}\right] = 0.$$

$$\frac{dr}{d\theta} - r\sin\theta + \cos\theta\frac{dr}{d\theta} = 0.$$

$$(1+\cos\theta)\frac{dr}{d\theta} - r\sin\theta = 0.$$

Replace $\frac{dr}{d\theta} = -r^2\frac{d\theta}{dr}$

$$(1+\cos\theta)\left(-r^{\cancel{2}}\frac{d\theta}{dr}\right) = \cancel{r}\sin\theta.$$

$$-\frac{1+\cos\theta}{\sin\theta}\,d\theta = \frac{1}{r}dr.$$

$$-(\mathrm{cosec}\,\theta + \cot\theta)d\theta = \frac{1}{r}dr$$

$$-\int\mathrm{cosec}\,\theta\,d\theta - \int\cot\theta.d\theta = \int\frac{1}{r}dr$$

$$-\log(\mathrm{cosec}\,\theta - \cot\theta) - \log(\sin\theta) = \log r + \log c.$$

$$-\left[\log\frac{(\mathrm{cosec}\,\theta - \cot\theta)}{\cancel{\sin\theta}}(\sin\theta)\right] = \log r + \log c$$

$$\log\left(\frac{\mathrm{cosec}\,\theta - \cot\theta}{\cancel{\sin\theta}}\right)^{-1}\frac{1}{\sin\theta} = \log(r.c)$$

$$\frac{1}{(\sin\theta)}\frac{\cancel{\sin\theta}\,1}{(\mathrm{cosec}\,\theta - \cot\theta)} = c.r.$$

$$\frac{1}{\sin\theta}\cdot\frac{1}{\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}} = cr.$$

$$\left(\frac{1}{\sin\theta}\right)\frac{1}{\frac{1-\cos\theta}{\sin\theta}} = cr.$$

$$cr = \frac{1}{1-\cos\theta}$$

(or)

$$\frac{1}{cr} = 1-\cos\theta$$

$$\frac{1}{r} = c(1-\cos\theta)$$

(6) $r = a(1-\cos\theta)$

Sol:- $r = a(1-\cos\theta) \rightarrow ①$

diff. w.r.to 'θ'

$$\frac{dr}{d\theta} = a(0-(-\sin\theta))$$

$$\frac{dr}{d\theta} = a\cdot\sin\theta.$$

$$a = \frac{1}{\sin\theta}\cdot\frac{dr}{d\theta}$$

from ①, $r = \frac{1}{\sin\theta}\cdot\frac{dr}{d\theta}(1-\cos\theta)$

Replace $\frac{dr}{d\theta} = -r^2\frac{d\theta}{dr}$.

$$\cancel{r} = \frac{1}{\sin\theta}\cdot(-r\cancel{^2})\frac{d\theta}{dr}(1-\cos\theta)$$

$$\frac{1}{r}\,dr = \frac{-(1-\cos\theta)}{\sin\theta}\,d\theta$$

$$-\frac{1}{r}\,dr = (\operatorname{cosec}\theta - \cot\theta)\,d\theta$$

$$-\int\frac{1}{r}\,dr = \int\operatorname{cosec}\theta\,d\theta - \int\cot\theta\,d\theta.$$

$$-\log r = \log(\operatorname{cosec}\theta-\cot\theta) - \log(\sin\theta) + \log c$$

$$-\log r - \log c = \log\left(\frac{\operatorname{cosec}\theta-\cot\theta}{\sin\theta}\right)$$

$$-[\log(c.r)] = \log(\operatorname{cosec}^2\theta - \operatorname{cosec}^2\theta.\cos\theta)$$

$$\log(cr)^{-1} = \log \operatorname{cosec}^2\theta\,(1-\cos\theta)$$

$$\frac{1}{cr} = \operatorname{cosec}^2\theta\,(1-\cos\theta)$$

$$\frac{1}{r} = c\cdot\operatorname{cosec}^2\theta\,(1-\cos\theta)$$

(7) $r = a(1+\sin^2\theta)$

Sol:- $r = a(1+\sin^2\theta) \rightarrow ①$

diff. w. r. to 'θ'

$$\frac{dr}{d\theta} = a(0 + 2\sin\theta.\cos\theta)$$

$$\frac{dr}{d\theta} = 0 + 2a.\sin\theta.\cos\theta$$

$$\frac{dr}{d\theta} = a.\sin 2\theta$$

$$a = \frac{1}{\sin 2\theta}.\frac{dr}{d\theta}$$

from ①,

$$r = \frac{1}{\sin 2\theta}.\frac{dr}{d\theta}(1+\sin^2\theta)$$

Replace $\frac{dr}{d\theta} = -r^2\frac{d\theta}{dr}$

$$\cancel{r} = \frac{1}{\sin 2\theta}(-r^{\cancel{2}})\frac{d\theta}{dr}(1+\sin^2\theta)$$

$$\frac{1}{r}dr = -\left(\frac{1+\sin^2\theta}{\sin 2\theta}\right)d\theta$$

$$-\frac{1}{r}dr = \left(\frac{1}{\sin 2\theta} + \frac{\sin^{\cancel{2}}\theta}{2\sin\cancel{\theta}\cos\theta}\right)d\theta$$

$$-\frac{1}{r}dr = \left(\mathrm{cosec}\,2\theta + \frac{1}{2}\tan\theta\right).d\theta$$

$$-\int\frac{1}{r}dr = \int\mathrm{cosec}\,2\theta\,d\theta + \frac{1}{2}\int\tan\theta\,d\theta$$

$$-\log r = \frac{\log(\mathrm{cosec}\,2\theta - \cot 2\theta)}{2} + \frac{1}{2}.\log(\sec\theta) + \log c$$

$$-\log r - \log c = \frac{1}{2}[\log(\mathrm{cosec}\,2\theta - \cot 2\theta)(\sec\theta)].$$

$$-2[\log r + \log c] = \log[(\mathrm{cosec}\,2\theta - \cot 2\theta)(\sec\theta)]$$

$$-2\log(rc) = \log(\mathrm{cosec}\,2\theta - \cot 2\theta)(\sec\theta)$$

$$\cancel{\log}(r.c)^{-2} = \cancel{\log}[(\mathrm{cosec}\,2\theta - \cot 2\theta)(\sec\theta)]$$

$$\frac{1}{r^2c^2} = \left(\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta}\right)\sec\theta.$$

$$\frac{1}{r^2c} = \left(\frac{1-\cos 2\theta}{\sin 2\theta}\right).\sec\theta.$$

$$\frac{1}{r^2c} = \frac{\cancel{2}\sin^{\cancel{2}}\theta}{\cancel{2}\cancel{\sin\theta}.\cos\theta}.\frac{1}{\cos\theta}$$

$$\frac{1}{r^2} = c.\sec\theta.\tan\theta.$$

(8) $r^2 = a^2 \sin 2\theta$

Sol:- $\log r^2 = \log(a^2 \cdot \sin 2\theta)$

$2\log r = \log a^2 + \log \sin 2\theta$

$2\log r = 2 \cdot \log a + \log \sin 2\theta$

diff w. r. to 'θ'

$2 \cdot \frac{1}{r} \cdot \frac{dr}{d\theta} = 0 + \frac{1}{\sin 2\theta}(\cos 2\theta) 2$

$\cancel{2} \cdot \frac{1}{r} \frac{dr}{d\theta} = \cot 2\theta \cdot \cancel{2}$

Replace $\frac{dr}{d\theta} = -r^2 \frac{d\theta}{dr}$

$\frac{1}{\cancel{r}}(-r^{\cancel{2}}) \frac{d\theta}{dr} = \cot 2\theta$

$-\frac{1}{\cot 2\theta} d\theta = \frac{1}{r} dr$

$-\int \tan 2\theta \cdot d\theta = \int \frac{1}{r} dr$

$-\frac{\log(\sec 2\theta)}{2} = \log r + \log C$

$-\frac{1}{2}\log(\sec 2\theta) = \log r + \log c$

$\cancel{\log}(\sec 2\theta)^{-1/2} = \cancel{\log}(C \cdot r)$

$\sec 2\theta = (C r)^{-2}$

$\sec 2\theta = \frac{1}{C^2 r^2}$

$C \cdot \sec 2\theta = \frac{1}{r^2}$

(9) $r = a \cdot \cos^2\theta$

Sol:- $r = a \cdot \cos^2\theta \rightarrow (1)$

diff w. r. to 'θ'

$\frac{dr}{d\theta} = a \cdot 2\cos\theta(-\sin\theta)$

$\frac{dr}{d\theta} = a \cdot -(\sin 2\theta)$

$a = \frac{-1}{\sin 2\theta} \cdot \frac{dr}{d\theta}$

from (1), $r = \frac{-1}{\sin 2\theta} \cdot \frac{dr}{d\theta} \cdot \cos^2\theta$

Replace $\frac{dr}{d\theta} = -r^2 \frac{d\theta}{dr}$

$$\cancel{\gamma} = \frac{-1}{\sin 2\theta} \cdot (-r\cancel{^2}) \frac{d\theta}{dr} (\cos^2\theta)$$

$$\frac{1}{r} dr = \frac{\cos^2\theta}{\sin 2\theta} \cdot d\theta$$

$$\frac{1}{r} dr = \frac{\cos^{\cancel{2}}\theta}{2\sin\theta \cdot \cancel{\cos\theta}} d\theta.$$

$$\frac{1}{r} dr = \frac{1}{2} \cot\theta \cdot d\theta.$$

$$\int \frac{1}{r} dr = \frac{1}{2} \int \cot\theta \, d\theta.$$

$$\log r = \frac{1}{2} \log (\sin\theta) + \log c.$$

$$\log r - \log c = \frac{1}{2} \log(\sin\theta)$$

$$2 \log \left(\frac{r}{c}\right) = \log(\sin\theta)$$

$$\cancel{\log} \frac{r^2}{c^2} = \cancel{\log} (\sin\theta)$$

$$r^2 = c \cdot \sin\theta.$$

(10) $r = 2a(\sin\theta + \cos\theta)$

Sol:- $r = 2a(\sin\theta + \cos\theta) \to ①$

diff. w. r. to 'θ'.

$$\frac{dr}{d\theta} = 2a \cdot (\cos\theta - \sin\theta)$$

$$2a = \frac{1}{\cos\theta - \sin\theta} \cdot \frac{dr}{d\theta}.$$

from ①,

$$r = \frac{1}{\cos\theta - \sin\theta} \cdot \frac{dr}{d\theta} (\sin\theta + \cos\theta)$$

Replace $\frac{dr}{d\theta} = -r^2 \frac{d\theta}{dr}$.

$$\cancel{r} = \frac{1}{\cos\theta - \sin\theta} (-r\cancel{^2}) \frac{d\theta}{dr} (\sin\theta + \cos\theta)$$

$$\frac{1}{r} dr = \frac{\cancel{-}(\sin\theta + \cos\theta)}{\cancel{-}(-\cos\theta + \sin\theta)} d\theta.$$

$$\int \frac{1}{r} dr = \int \frac{\sin\theta + \cos\theta}{-\cos\theta + \sin\theta} \cdot d\theta.$$

$$\log r = \log(\sin\theta - \cos\theta) + \log c.$$

$$\cancel{\log} r = \cancel{\log} (\sin\theta - \cos\theta) \cdot c$$

$$r = c \cdot (\sin\theta - \cos\theta)$$

Thursday 10/10

Law of Natural Decay & Growth:

(4) In a certain culture of bacteria the rate of increases is proportional to the number present.

(a) If it is found that the number doubles in 4 hrs, How many may expected at the end of 12 hrs.

Sol: We have, $y = c \cdot e^{kt} \rightarrow (1)$

Initially $t = 0$ and $y = y_0$

from (1),

$$y_0 = c \cdot e^{k(0)}$$

$$= c \cdot e^{0}$$

$$y_0 = c(1)$$

$$\boxed{c = y_0}$$

$$y = y_0 e^{kt} \rightarrow (2)$$

$t = 4$ hrs and $y = 2y_0$

$$2\cancel{y_0} = \cancel{y_0} e^{k(4)}$$

$$e^{4k} = 2.$$

$$4k = \log 2.$$

$$k = \frac{1}{4} \cdot \log 2$$

$$\boxed{k = 0.17329}$$

$$y = y_0 e^{(0.17329)t} \rightarrow (3)$$

And also $t = 12$, $y = ?$

$$y = y_0 e^{(0.17329)12}$$

$$y = y_0 (8.0003076)$$

$$\boxed{y = 8y_0.}$$

(6)

Sol:- We have $y = ce^{Kt} \longrightarrow (1)$

Initially $t=0$ and $y=y_0$

from (1), $y_0 = c\,e^{K(0)}$

$= c \cdot e^0$

$y_0 = c(1)$

$\Rightarrow \boxed{c = y_0}$

$y = y_0 e^{Kt} \longrightarrow (2)$

$t = 5$ hrs and $y = 3y_0$.

$3\cancel{y_0} = \cancel{y_0}\, e^{K(5)}$

$e^{5k} = 3$

$5k = \log 3$.

$k = \frac{1}{5} \log 3$

$\boxed{k = 0.21972}$

$y = y_0 \cdot e^{(0.21972)t} \longrightarrow (3)$

(a)

And also $t = 10$ hrs and $y = ?$

$y = y_0\, e^{(0.21972)10}$.

$y = y_0\,(8.999778807)$

$\boxed{y = 9y_0.}$

(b) $t = ?$ and $y = 10y_0$

$$10\cancel{y_0} = \cancel{y_0} \cdot e^{(0.21972)t}$$

$$e^{(0.21972)t} = 10.$$

$$(0.21972)t = \log 10$$

$$t = \frac{1}{0.21972} \log 10.$$

$$t = 10.4796335.$$

$$\boxed{t = 10.48.} \text{ hrs.}$$

$$\boxed{t = 11} \text{ hrs.}$$

(10) The rate of at which the bacteria multiply is proportional to the instantaneous number present. If the original number doubles in 2 hrs. In how many hours will it triple.

sol: We have $y = ce^{Kt} \rightarrow ①$

Initially $t = 0$ and $y = y_0$.

from ①,

$$y_0 = ce^{K(0)}$$

$$= ce^{0}$$

$$y_0 = c(1)$$

$$\Rightarrow \boxed{c = y_0}$$

$$y = y_0 e^{Kt} \rightarrow ②$$

$t = 2$ hrs and $y = 2y_0$.

$$2\cancel{y_0} = \cancel{y_0} \cdot e^{K(2)}$$

$$e^{2K} = 2$$

$$2K = \log 2$$

$$K = \frac{1}{2} \log 2.$$

$$K = 0.34657$$

$$\boxed{K = 0.3466}$$

$$y = y_0 \cdot e^{(0.3466)t} \rightarrow ③$$

And also $t = ?$ $\quad y = 3y_0$.

$$3y_0 = y_0 e^{(0.3466)t}$$

$$e^{(0.3466)t} = 3$$

$$(0.3466)t = \log 3$$

$$t = \frac{1}{0.3466} \log 3$$

$$t = 3.169683$$

$\boxed{t \cong 3}$ hrs.

* The world population at the begining ~~1970~~ 1970 was 3.6 billion.

(a) The weight of the earth is 6.586×10^{21} tones. If the population continuous to increase exponentially with a growth constant $K = 0.02$ and with time measure in years, In what year did the weight of the people equal to the weight of the Earth, If we assume that the average person weight is 120 found.

decay

* (b) In a certain chemical reaction the rate of conversion of a substance, at time 't' is proportional to the quantity of the substance still untransformed at that instant. At the end of '1' hour 60 grams remain and at the end of '4' hours 21 grams. How many grams of the 1st substance was there initially.

Sol:- Wehave by law of natural growth

$$y = c \cdot e^{Kt} \rightarrow ①$$

Initially $t = 0$ and $y = 3.6 \times 10^9$

$$3.6 \times 10^9 = c \cdot e^{K(0)}$$

$$3.6 \times 10^9 = c\, e^{(0)}$$

$$\boxed{c = 3.6 \times 10^9}$$

$$y = 3.6 \times 10^9 e^{Kt} \rightarrow ②$$

Given that $k = 0.02$.

$y = 3.6 \times 10^9 \, e^{(0.02)t}$ $\rightarrow$ ③

Weight of the Earth 6.586×10^{21} tones.

Weight of the people $3.6 \times 10^9 \, e^{(0.02)t} \times 120$ pounds

(1 ton = 2240 pounds)

weight of the Earth $6.586 \times 10^{21} \times 2240$. pounds.

weight of the people = weight of the Earth

$3.6 \times 10^9 e^{(0.02)t} \times 120 = 6.586 \times 10^{21} \times 2240$

$$e^{(0.02)t} = \frac{6.586 \times 10^{\cancel{21}\,12} \times 224\cancel{0}}{3.6 \times \cancel{10^9} \times 12\cancel{0}}$$

$$= \frac{6.586 \times 10^{12} \times 224}{43.2} = \frac{1.475264 \times 10^{13}}{43.2}$$

$e^{(0.02)t} = 3.414962963 \times 10^{13}$

$(0.02)t = \log (3.414962963 \times 10^{13})$

$(0.02)t = 31.16177286$

$$t = \frac{31.16177286}{0.02}$$

$t = 1558.088643$

at $t = 1558 + 1970$

$= 3528$ year.

The rate of the population and weight of the earth are equal.

1.

We have $y = c \cdot e^{kt}$ $\rightarrow$ ①

Initially $t = 0$ and $y = 100$

from ①, $100 = c \cdot e^{k(0)}$

$= c \cdot e^0$

$= c(1)$

$\Rightarrow \boxed{c = 100}$

$$y = 100 \cdot e^{Kt} \rightarrow ②$$

$t = 1$ and $y = 332$

$$332 = 100 \cdot e^{K(1)}$$

$1\frac{1}{2}$

$3/2$

$$e^K = \frac{332}{100}$$

$$e^K = 3.32$$

$$\boxed{K = \log(3.32)}$$

$$K = 1.19996$$

$$y = 100 \cdot e^{(1.19996)t} \rightarrow ③$$

And also. $t = 1\frac{1}{2}$ hour and $y = ?$

$$y = 100 \cdot e^{(1.19996)\,1.5}$$

$$y = 100 \times 6.0492$$

$$y = 604.92 \approx 605.$$

(2) We have $y = c \cdot e^{kt} \rightarrow ①$

Initially $t = 0$ and $y = y_0$

$$y_0 = c\,e^{K(0)}$$

$$= c \cdot e^0$$

$$y_0 = c(1)$$

$$\boxed{c = y_0}$$

from①,

$$y = y_0 e^{Kt} \rightarrow ②$$

$t = 2$ and $y = 2y_0$.

$$2y_0 = y_0\,e^{K(2)}$$

$$e^{2K} = 2$$

$$2K = \log 2$$

$$\boxed{K = \tfrac{1}{2}\log 2}$$

$y = y_0 \cdot e$

$\boxed{K = 0.34657}$

$$y = y_0 \cdot e^{(0.34657)t} \rightarrow ③$$

And also $t = 8$ and $y = ?$

$$y = y_0 \cdot e^{(0.34657)8}$$

$$y = y_0(15.9995)$$

$$\boxed{y \cong 16 y_0}$$

And also $t = ?$ and $y = 8y_0$.

$$8\cancel{y_0} = \cancel{y_0} \cdot e^{(0.34657)t}$$

$$e^{(0.34657)t} = 8$$

$$(0.34657)t = \log 8$$

$$t = \frac{1}{0.34657} \log 8$$

$$t = 6.0000602157.$$

$$t \cong 6.1.$$

$$\boxed{t \cong 6 \text{ hours}}$$

(5).

We have $y = ce^{kt} \rightarrow ①$

Initially $t = 0$ and $y = y_0$

$$y_0 = ce^{k(0)}$$
$$= c \cdot e^0$$
$$= c(1)$$
$$\Rightarrow \boxed{c = y_0}$$

from ①,

$$y = y_0 e^{kt} \rightarrow ②$$

$t = 50$ and $y = 2y_0$.

$$2\cancel{y_0} = \cancel{y_0} \cdot e^{(50)k}.$$

$$e^{K(50)} = 2$$
$$K(50) = \log 2$$
$$K = \frac{1}{50}\log 2$$
$$\boxed{K = 0.01386}$$
$$y = y_0 e^{(0.01386)t} \rightarrow ②$$

And also $t = ?$ and $y = 3y_0$.

$$3\cancel{y_0} = \cancel{y_0} \cdot e^{(0.01386)t}$$
$$e^{(0.01386)t} = 3$$
$$(0.01386)t = \log 3$$
$$t = \frac{1}{0.01386}\log 3$$
$$t = 79.2649$$
$$\boxed{t \cong 79 \text{ years}}$$

(8)

We have $y = ce^{kt} \rightarrow ①$

Initially $t=0$ and $y = y_0$

$$y_0 = ce^{K(0)}$$
$$= c \cdot e^0$$
$$= c(1)$$
$$\boxed{c = y_0}$$

from ①, $y = y_0 e^{kt} \rightarrow ②$

And $t = 3$. and $y = 2y_0$.

$$2\cancel{y_0} = \cancel{y_0} e^{K(3)}$$
$$e^{K(3)} = 2$$
$$K(3) = \log 2$$
$$K = \frac{1}{3}\log 2$$
$$\boxed{K = 0.23104}$$

$$y = y_0 \cdot e^{(0.23104).t} \rightarrow ③$$

And also $t = 15$ and $y = ?$

$$y = y_0 \cdot e^{(0.23104)15}$$

$$y = 31.99565\, y_0$$

$$\boxed{y \cong 32\, y_0}$$

(9)

We have $y = c \cdot e^{Kt} \rightarrow ①$

Initially $t = 0$ and $y = 100$.

$$100 = c e^{K(0)}$$

$$100 = c \cdot e^{(0)}$$

$$100 = c \cdot (1)$$

$$\boxed{c = 100}$$

from ①, $y = 100 e^{Kt} \rightarrow ②$

And $t = 12$ hours and $y = 400$.

$$4\cancel{00} = 1\cancel{00} \cdot e^{K(12)}$$

$$e^{K(12)} = 4$$

$$K(12) = \log 4$$

$$K = \frac{1}{12} \log 4$$

$$\boxed{K = 0.115524}$$

$$y = 100\, e^{(0.115524)t} \rightarrow ③$$

And also $t = 3$ and $y = ?$

$$y = 100\, e^{(0.115524)3}$$

$$y = 100 \times 1.41421$$

$$y = 141.421 \rightarrow \boxed{y \approx 141.}$$

Law of Natural Decay:

(4)

We have the law of natural decay is $y = ce^{-kt} \rightarrow (1)$

Initially $t=0$, $y=y_0$

$y_0 = ce^{-k(0)}$

$y_0 = ce^{(0)}$

$y_0 = c\,(1)$

$\Rightarrow \boxed{c = y_0}$

$y = y_0 e^{-kt} \rightarrow (2)$

$t=1500$ and $y = \frac{y_0}{2}$

$\frac{\cancel{y_0}}{2} = \cancel{y_0} e^{-k(1500)}$

$\frac{1}{2} = e^{-k(1500)}$

$e^{-k(1500)} = 0.5$

$-k(1500) = \log(0.5)$

$k = \frac{-1}{1500} \log(0.5)$

$k = -(-4.620981204 \times 10^{-4})$

$k = 0.0004620981204$

$k = 0.000462$

$y = y_0 e^{-(0.000462)t} \rightarrow (3)$

(a) $t = 4500$ and $y = ?$

$y = y_0 e^{-(0.000462)(4500)}$

$y = y_0 (0.125055204)$

$y = 0.125\, y_0$

$y = 12.5\, y_0$.

(b) $t = ?$ and $y = \frac{1}{10} y_0$.

$\frac{1}{10} \cancel{y_0} = \cancel{y_0} \cdot e^{-(0.000462)t}$

$e^{-(0.000462)t} = 0.1$

$-(0.000462)t = \log 0.1$

$t = \frac{-1}{0.000462} \log(0.1)$

$= -(-4983.950418)$

$= 4983.950418$

$\boxed{t \simeq 4984}$ years.

(1) In a chemical reaction the rate of conversion of a substance at time 't' is proportional to

By law of natural decay,

we have $y = ce^{-kt} \rightarrow ①$

Initially $t = 0$ and $y = y_0$.

$y_0 = ce^{-k(0)}$

$= c \cdot e^{(0)}$

$y_0 = c(1)$

$\Rightarrow \boxed{c = y_0}$

$y = y_0 e^{-kt} \rightarrow ②$

And $t = 1$ and $y = 60$ grams

~~and~~ $60 = y_0 \, e^{-k(1)}$

$60 = y_0 \cdot e^{-k} \rightarrow ③$

And also $t = 4$ and $y = 21$ grams

$21 = y_0 e^{-k(4)}$

$21 = y_0 \cdot e^{-4k} \rightarrow ④$

divide ③/④

$\Rightarrow \frac{\cancel{y_0} e^{-k}}{\cancel{y_0} \cdot e^{-4k}} = \frac{\cancel{60}\, 20}{\cancel{21}\, 7}$

$\frac{1}{e^{-3k}} = \frac{20}{7}$

$e^{3K} = \frac{20}{7}$

$e^{3K} = 2.857142857.$

$3K = \log(2.857)$

$K = \frac{1}{3}\log(2.857)$

$K = 0.349924$

$\boxed{K = 0.3499}$

sub 'k' value in Eqn ③.

from ③, $60 = y_0 e^{-K}$

$60 = y_0 e^{-(0.3499)}$

$y_0 = \frac{1}{e^{-(0.3499)}} \cdot 60.$

$y_0 = e^{(0.3499)} \cdot 60.$

$y_0 = 85.1355$

$\boxed{y_0 \cong 85 \text{ grams}}$

- If 30% of a radio active substance desappear in 10 days. How long will it take for 90% of its to disappear.

Sol: We have $y = ce^{-Kt} \rightarrow ①$

Initially $t=0$ and $y = y_0$

$y_0 = ce^{-K(0)}$

$= c \cdot e^{(0)}$

$= c(1)$

$\Rightarrow \boxed{c = y_0}$

$y = y_0 e^{-Kt} \rightarrow ②$

And $t = 10$ and $y = 70\%\, y_0$

$= \frac{70}{100} y_0$

$\frac{70}{100} \cancel{y_0} = \cancel{y_0} e^{-K(10)}$

$e^{-10K} = 0.7$

$$-10K = \log 0.7$$

$$K = \frac{-1}{10}\log(0.7)$$

$$K = 0.0356674994$$

$$\boxed{K = 0.0357}$$

$$y = y_0 \cdot e^{-(0.0357)t} \rightarrow (3).$$

And also $t = ?$ and $y = 10\%\ y_0$.

$$= \frac{10}{100} y_0$$

$$\frac{10}{100} y_0 = y_0\, e^{-(0.0357)t}$$

$$e^{-(0.0357)t} = 0.1$$

$$-(0.0357)t = \log 0.1$$

$$t = \frac{-1}{0.0357}\log(0.1)$$

$$= -(-64.49818188)$$

$$t = 64.5$$

$$\boxed{t \cong 64 \text{ days}}$$

(3) Find the half-life of uranium, which disintegrates at a rate proportional to the amount present at any instant given that m_1 and m_2 grams of Uranium are present at t_1 and t_2 respectively.

Sol: We have $y = ce^{-Kt} \rightarrow (1)$

Initially $t = 0$, $y = M$.

$$M = ce^{-K(0)}$$

$$= c \cdot e^{(0)}$$

$$M = c.(1)$$

$$\boxed{c = M}$$

$$y = M e^{-Kt} \rightarrow (2)$$

And $t = t_1$ and $y = m_1$, $t = t_2$ and $y = m_2$

$$m_1 = M e^{-Kt_1} \rightarrow ③$$

$$m_2 = M e^{-Kt_2} \rightarrow ④$$

$$\frac{③}{④} \Rightarrow \frac{\cancel{M} e^{-Kt_1}}{\cancel{M} e^{-Kt_2}} = \frac{m_1}{m_2}$$

$$\frac{e^{-Kt_1}}{e^{-Kt_2}} = \frac{m_1}{m_2}$$

$$e^{-Kt_1} \cdot e^{+Kt_2} = \frac{m_1}{m_2}$$

$$e^{Kt_2 - Kt_1} = \frac{m_1}{m_2}$$

$$e^{K(t_2 - t_1)} = \frac{m_1}{m_2}$$

$$K(t_2 - t_1) = \log \frac{m_1}{m_2}$$

$$\boxed{K = \frac{1}{t_2 - t_1} \log \frac{m_1}{m_2}}$$

Sub 'K' in Equn ②.

$$y = M \cdot e^{-\left[\left(\frac{1}{t_2 - t_1}\right) \log \frac{m_1}{m_2}\right] t} \rightarrow ⑤$$

And also $t = ?$ and $y = \frac{M}{2}$

$$\frac{\cancel{M}}{2} = \cancel{M} \cdot e^{-\left(\frac{\log \frac{m_1}{m_2}}{t_2 - t_1}\right) t}$$

$$e^{-\left(\frac{\log m_1/m_2}{t_2 - t_1}\right) t} = \frac{1}{2}$$

$$-\left(\frac{\log \frac{m_1}{m_2}}{t_2 - t_1}\right) t = \log 0.5$$

$$t = \frac{-1}{\frac{\log \frac{m_1}{m_2}}{t_2 - t_1}} \log 0.5$$

$$t = \frac{-(t_2 - t_1)}{\log \frac{m_1}{m_2}} \log 0.5$$

$$t = \frac{t_1 - t_2}{\log \frac{m_1}{m_2}} \log 0.5$$

(OR)

$$t = \frac{(t_1 - t_2)\log\left(\frac{1}{2}\right)}{\log \frac{m_1}{m_2}}$$

$$t = \frac{(t_1 - t_2)\log(1) - \log(2)}{\log \frac{m_1}{m_2}}$$

$$= \frac{(t_1 - t_2)(0 - \log 2)}{\log \frac{m_1}{m_2}}$$

$$= \frac{(t_1 - t_2)(-\log 2)}{\log \frac{m_1}{m_2}}$$

$$= \frac{(t_2 - t_1)\log 2}{\log \frac{m_1}{m_2}}$$

(2)

We have, $y = Ce^{-kt} \rightarrow ①$

Initially $t=0$ and $y = y_0$.

$$y_0 = Ce^{-k(0)}$$

$$y_0 = C\, e^{(0)}$$

$$= C(1)$$

$$\Rightarrow \boxed{C = y_0}$$

$$y = y_0 e^{-kt} \rightarrow ②$$

(5) We have $y = c\cdot e^{-Kt} \rightarrow ①$

Initially $t=0$ and $y=10$.

$$10 = c\cdot e^{-K(0)}$$

$$10 = c\, e^{(0)}$$

$$= c(1)$$

$$\Rightarrow \boxed{c=10}$$

$$y = 10\, e^{-Kt} \rightarrow ②$$

and $t=1$ and $y=0.051$

$$0.051 = 10\, e^{-K(1)}$$

$$\frac{0.051}{10} = e^{-K}$$

$$-K = \log\left(\frac{0.051}{10}\right)$$

$$K = -\log\left(\frac{0.051}{10}\right)$$

$$K = -(-5.278514739)$$

$$\boxed{K = 5.279}$$

$$y = 10e^{-(5.279)t} \rightarrow ③$$

And also $y=5$ and $t=?$

$$\cancel{5} = \cancel{10}^{2}\cdot e^{-(5.279)t}$$

$$1/2 = e^{-(5.279)t}$$

$$\frac{1}{2} = e^{-(5.279)t}$$

$$e^{-(5.279)t} = \frac{1}{2}$$

$$-(5.279)t = \log(1/2)$$

$$t = \frac{-1}{5.279} \log(1/2)$$

$$t = -(-0.131302743)$$

$$\boxed{t = 0.1313}$$

(2) Tuesday 15/10.

Newton's Law of Coolinge

(3)

By Newton's Law of cooling,

we have $T = T_A + c.e^{-kt} \rightarrow (1)$

Initially $t=0$, $T = 100\,°C$ and $T_A = 40°C$.

$$100 = 40 + ce^{-k(0)}$$

$$100 - 40 = ce^{(0)}$$

$$60 = c(1)$$

$$\Rightarrow \boxed{c = 60}$$

from (1),

~~100 = 40.~~

$$T = 40 + 60e^{-kt} \rightarrow (2)$$

$t = 4$, $T = 60$

$$60 = 40 + 60e^{-k(4)}$$

$$60 - 40 = 60 \cdot e^{-4k}$$

$$\cancel{20} = \overset{3}{\cancel{60}} e^{-4k}$$

$$\frac{1}{3} = e^{-4k}$$

$$-4k = \log \frac{1}{3}$$

$$k = \frac{-1}{4} \log \frac{1}{3}$$

$K = \frac{-1}{4} \log (0.33)$

$K = -(-0.277165656)$

$\boxed{K = 0.2772}$

$T = 40 + 60e^{-(0.2772)t} \rightarrow ③$

And also $t = ?$ $T = 50$

$50 = 40 + 60e^{-(0.2772)t}$

$1\cancel{0} = 6\cancel{0}e^{-(0.2772)t}$

$\frac{1}{6} = e^{-(0.2772)t}$

$-(0.2772)t = \log 1/6$

$t = \frac{-1}{0.2772} \log(1/6)$

$t = -(-6.41044242)$

$\boxed{t = 7 \text{ min}}$

By Newton's Law of Cooling,

we have $T = T_A + ce^{-Kt} \rightarrow ①$

Initially $t = 0$, $T = 370K$ and $T_A = 300K$.

$370 = 300 + ce^{-K(0)}$

$70 = ce^{(0)}$

$70 = c(1)$

$\boxed{C = 70}$

from ①, $T = 300 + 70e^{-Kt} \rightarrow ②$

And $t = 15$ min, $T = 340K$

$340 = 300 + 70e^{-K(15)}$

$4\cancel{0} = 7\cancel{0}e^{-15K}$

$4/7 = e^{-15K}$

$\cancel{4/7 = 0.6}$

$-15K = \log 4/7$

$K = \frac{-1}{15} \log(0.6)$

$K = \cancel{-(-0.0340\ldots)}$ 0.037307719

$\cancel{K = 0.034}$ $\boxed{K = 0.0373}$

$$T = 300 + 70e^{-(0.0373)t} \rightarrow (3)$$

And also $t = ?$ and $T = 310\,K$

$$310 = 300 + 70e^{-(0.0373)t}$$

$$1\cancel{0} = 7\cancel{0}e^{-(0.0373)t}$$

$$-(0.0373)t = \log(1/7)$$

$$t = \frac{-1}{0.0373}\log(1/7)$$

$$t = -(-52.1691729)$$

$$\boxed{t \cong 52\text{ min.}}$$

(7)

By Newton's Law of Cooling,

we have $T = T_A + ce^{-kt} \rightarrow (1)$

Initially $t=0$, $T = 100°c$ and $T_A = 25°C$.

$$100 = 25 + ce^{-k(0)}$$

$$75 = ce^{(0)}$$

$$75 = c(1)$$

$$\boxed{c = 75}$$

from (1), $T = 25 + 75e^{-kt} \rightarrow (2)$

$t = 10$ and $T = 80°c$

$$80 = 25 + 75e^{-k(10)}$$

$$80 - 25 = 75e^{-10k}$$

$$\overset{11}{\cancel{55}} = \overset{15}{\cancel{75}}e^{-10k}$$

$$11/15 = e^{-10k}$$

$$-10k = \log 11/15$$

$$k = \frac{-1}{10}\log(11/15)$$

$$k = -(-0.031015492)$$

$$\boxed{k = 0.031}$$

$$T = 25 + 75e^{-(0.031)t} \rightarrow (3)$$

~~Ans~~

(i) $t = 20$ and $T = ?$

$$T = 25 + 75e^{-(0.031)20}$$

$$T = 25 + 75\cdot(0.5379444\text{87})$$

$$T = 25 + 40.34583282$$

$$T = 25 + 40.346$$

$$T = 65.346$$

$$\boxed{T \cong 65^\circ c}$$

(ii) $t = ?$ and $T = 40^\circ c$

$$40 = 25 + 75e^{-(0.031)t}$$

$$\cancel{15} = \overset{5}{\cancel{75}}e^{-(0.031)t}$$

$$1/5 = e^{-(0.031)t}$$

$$-(0.031)t = \log(1/5)$$

$$t = \frac{-1}{0.031}\log(1/5)$$

$$t = -(-51.91\cancel{735201})$$

$$\boxed{t \cong 52 \text{ min}}$$

(8) By Newton's Law of Cooling,

we have $T = T_A + ce^{-Kt} \rightarrow ①$

Initially $t=0$, $T = 80^\circ c$ and $T_A = 30^\circ c$.

$$80 = 30 + ce^{-K(0)}$$

$$50 = ce^{(0)}$$

$$50 = c(1)$$

$$\Rightarrow \boxed{c = 50}$$

from ①, $T = 30 + 50e^{-Kt} \rightarrow ②$

and $t = 12$, $T = 60^\circ c$.

$$60 = 30 + 50e^{-K(12)}$$

$$60 - 30 = 50e^{-K(12)}$$

$$3\cancel{0} = 5\cancel{0}e^{-12k}$$

$$3/5 = e^{-12K}$$

$$-12K = \log 3/5$$

$$K = \frac{-1}{12} \log 3/5$$

(1) By Newton's Law of cooling,

we have $T = T_A + Ce^{-Kt} \rightarrow ①$

Initially $t = 0$, $T = 100°c$ and $T_A = 20°c$

$$100 = 20 + Ce^{-K(0)}$$

$$100 - 20 = Ce^{-(0)k}$$

$$80 = C \cdot e^{0}_{(1)}$$

$$\Rightarrow \boxed{C = 80}$$

from ①, $T = 20 + 80e^{-Kt} \rightarrow ②$

And $t = 10$, $T = 25°c$

$$25 = 20 + 80e^{-K(10)}$$

$$25 - 20 = 80e^{-10k}$$

$$\cancel{5} = \cancel{80}^{16} e^{-10k}$$

$$\frac{1}{16} = e^{-10k}$$

$$-10k = \log(1/16)$$

$$K = \frac{-1}{10} \log(1/16)$$

$$K = -(-0.27725887 2)$$

$$\boxed{K = 0.28}$$

$$T = 20 + 80 \cdot e^{-(0.28)t} \rightarrow ③$$

And also $t = \overset{(0.5)}{1/2}$ hr and $T = ?$

$$T = 20 + 80e^{-(0.28)(0.5)}$$

$$T = 20 + 80 \times 0.869358235$$

$$T = 20 + 69.54865883$$

$$T = 20 + 69$$

$$\boxed{T \cong 89°c}$$

~~And also~~

(2) By Newton's Law of cooling,

we have $T = T_A + Ce^{-Kt} \rightarrow (1)$

Initially $t = 0$, $T = 75°C$ and $T_A = 25°C$.

$$75 = 25 + Ce^{-K(0)}$$

$$75 - 25 = C \cdot e^{(0)}$$

$$50 = C(1)$$

$$\Rightarrow \boxed{C = 50}$$

from (1),

$$T = 25 + 50e^{-Kt} \rightarrow (2)$$

$t = 10$ min, $T = 65°C$.

$$65 = 25 + 50e^{-K(10)}$$

$$65 - 25 = 50e^{-10K}$$

$$4\cancel{0} = 5\cancel{0}e^{-10K}$$

$$-10K = \log(4/5)$$

$$K = \frac{-1}{10}\log(4/5)$$

$$K = -(-0.022314355)$$

$$\boxed{K = 0.0223.}$$

$$T = 25 + 50e^{-(0.0223)t} \rightarrow (3)$$

And also $t = 20$ min, $T = ?$

$$T = 25 + 50e^{-(0.0223)20}$$

$$T = 25 + 32.0091886$$

$$T = 25 + 32$$

$$\boxed{T \simeq 57}$$

And also $t = ?$ and $T = 55°C$

$$55 = 25 + 50e^{-(0.0223)t}$$

$$55 - 25 = 50e^{-(0.0223)t}$$

$$3\cancel{0} = 5\cancel{0}e^{-(0.0223)t}$$

$$t = \frac{-1}{0.0223}\log(3/5)$$

$$= -(-22.90697864)$$

$$\boxed{t \simeq 23}$$

(5) By Newton's Law of Cooling,

we have $T = T_A + ce^{-Kt} \rightarrow ①$

Initially $t=0$, $T = 100°C$; $T_A = 20°C$

$$100 = 20 + ce^{-K(0)}$$

$$100 - 20 = ce^{(0)}$$

$$80 = c(1)$$

$$\Rightarrow \boxed{C = 80}$$

from ①,

$$T = 20 + 80e^{-Kt} \rightarrow ②$$

$t = 1$ min, $T = 60°C$

$$60 = 20 + 80e^{-K(1)}$$

$$60 - 20 = 80e^{-K}$$

$$4\cancel{0} = \overset{2}{\cancel{80}}e^{-K}$$

$$1/2 = e^{-K}$$

$$-K = \log(1/2)$$

$$K = -\log(1/2)$$

$$K = -(-0.69314718)$$

$$\boxed{K = 0.693}$$

$$T = 20 + 80e^{-(0.693)t} \rightarrow ③$$

And also $t = 2$ min and $T = ?$

$$T = 20 + 80e^{-(0.693)2}$$

$$T = 20 + 20.00588809$$

$$T = 20 + 20$$

$$\boxed{T \cong 40}$$

(6) By Newton's Law of Cooling,

we have $T = T_A + ce^{-Kt} \rightarrow ①$

Initially $t=0$, $T = 100°C$ and $T_A = 30°C$

$$100 = 30 + ce^{-K(0)}$$

$$100 - 30 = c \cdot e^{(0)}$$

$$70 = c(1)$$

$$\boxed{c = 70}$$

from ①, $T = 30 + 70e^{-Kt} \rightarrow ②$

$t = 10$ min , $T = 80°c$

$$80 = 30 + 70e^{-K(10)}$$

$$80 - 30 = 70\, e^{-10K}$$

$$50 = 70\, e^{-10K}$$

$$e^{-10K} = 5/7$$

$$-10K = \log(5/7)$$

$$K = \frac{-1}{10}\log(5/7)$$

$$K = -(-0.033647223)$$

$$\boxed{K = 0.034}$$

$$T = 30 + 70e^{-(0.034)t} \rightarrow ③$$

And also $t = ?$ and $T = 40°c$

$$40 = 30 + 70 \cdot e^{-(0.034)t}$$

$$10 = 70 e^{-(0.034)t}$$

$$e^{-(0.034)t} = (1/7)$$

$$-(0.034)t = \log(1/7)$$

$$t = \frac{-1}{0.034}\log(1/7)$$

$$t = -(-57.23265144)$$

$$\boxed{t \simeq 57}$$

(9) By Newton's Law of Cooling,

we have $T = T_A + ce^{-Kt} \rightarrow ①$

Initially $t=0$, $T = 100$, $T_A = 15°c$.

$$100 = 15 + ce^{-K(0)}$$

$$85 = c\,e^{(0)}$$

$$85 = c(1)$$

$$\Rightarrow \boxed{c = 85}$$

$$T = 15 + 85e^{-Kt} \rightarrow ②$$

$t = 5\text{ min}$, $T = 60°C$

$$60 = 15 + 85\, e^{-k(5)}$$

$$45 = 85\, e^{-5k}$$

$$e^{-5k} = \frac{45}{85}$$

$$e^{-5k} = 0.5294117647$$

$$e^{-5k} = 0.53$$

$$-5k = \log(0.53)$$

$$k = \frac{-1}{5}\log(0.53)$$

$$k = -(-0.1269756509)$$

$$\boxed{k = 0.13}$$

$$T = 15 + 85\, e^{-(0.13)t} \rightarrow ③$$

And also $t = 5$, $T = ?$

$$T = 15 + 85\, e^{-(0.13)5}$$

$$T = 15 + 44.37389102$$

$$T = 15 + 44$$

$$\boxed{T \simeq 59}$$

(10) By Newton's Law of Cooling,

we have $T = T_A + c\, e^{-kt} \rightarrow ①$

Initially $t = 0$, $T = 110°c$, $T_A = 10°c$

$$110 = 10 + c\, e^{-k(0)}$$

$$100 = c\, e^{(0)}$$

$$100 = c(1)$$

$$\boxed{c = 100}$$

from ①, $T = 10 + 100\, e^{-kt} \longrightarrow ②$

$t = 1\text{ hr}$, $T = 60°c$

$$60 = 10 + 100\, e^{-k(1)}$$

$$\cancel{50} = \cancel{100}^{2}\, e^{-k}$$

$$e^{-k} = 1/2$$

$-k = \log(1/2)$

$k = -\log(1/2)$

$k = -(-0.69314718)$

$\boxed{k = 0.693}$

$T = 10 + 100e^{-(0.693)t} \rightarrow (3)$

And also $t = ?$ $T = 30°C$

$30 = 10 + 100e^{-(0.693)t}$

$\cancel{2}\cancel{0} = \cancel{10}\cancel{0}\,5\, e^{-(0.693)t}$

$1/5 = e^{-(0.693)t}$

$-(0.693)t = \log(1/5)$

$t = \frac{-1}{0.693}\log(1/5)$

$t = -(-2.32242123)$

$\boxed{t \cong 2\,hr}$

Tuesday

15/10 Electrical circuits:

(1) A constant electromotive force E volts is applied to a circuit containing a constant resistance 'R' ohms in series and a constant inductance 'N' henry's. If the initial current is '0'. Show that the current builds up to half of its maximum in $\frac{L\log 2}{R}$ sec.

(2) A resistance of 100 ohm's and inductance of 0.5 henry are connected in a series with a battery of 20 volts. Find the current in the circuit, if initially there is no current in the circuit.

(3) A voltage Ee^{-at} is applied at $t=0$ to a circuit containing inductance 'L' and resistance 'R'. Show that at any time t is $\frac{E}{R-aL}\left(e^{-at} - e^{-\frac{R}{L}t}\right)$.

(4) Solve the eqn $L\frac{di}{dt} + Ri = 200\cos(300t)$. when $R = 100$, $L = 0.05$. and find 'i'. Given that $i=0$ when $t=0$, what value thus 'i' approach after a long time.

① ~~By using Krichhoff's of~~

By using Kirchoff's Law the equ^n of the LR circuit is $L\frac{di}{dt} + Ri = E$

$$\frac{di}{dt} + \frac{Ri}{L} = \frac{E}{L} \longrightarrow ①$$

Equ^n ① is in linear form $\frac{dy}{dx} + Py = Q$

$$P = \frac{R}{L} \quad \text{and} \quad Q = \frac{E}{L}$$

I.F $e^{\int P(t)dt}$

$$= e^{\int \frac{R}{L} dt}$$

$$= e^{\frac{R}{L}\int (1)dt}$$

$$= e^{\frac{R}{L}t}$$

Now the solution of equ^n ① is

$$i \cdot e^{\frac{R}{L}t} = \int \frac{E}{L} e^{\frac{R}{L}t} + c$$

$$= \frac{E}{L} \int e^{\frac{R}{L}t} + c$$

$$= \frac{E}{\cancel{L}} \frac{e^{\frac{R}{L}t}}{R/\cancel{L}} + c$$

$$i \cdot e^{\frac{R}{L}t} = \frac{E}{R} e^{\frac{R}{L}t} + c$$

$$i \cdot \cancel{e^{R/L\,t}} = \cancel{e^{R/L\,t}}\left(\frac{E}{R} + c\, e^{-R/L\,t}\right)$$

$$i = \frac{E}{R} + c\, e^{-\frac{R}{L}t}$$

Initially $t=0$ and $i=0$

$$0 = \frac{E}{R} + c \cdot e^{-\frac{R}{L}(0)}$$

$$-\frac{E}{R} = c e^{(0)}$$

$$\frac{-E}{R} = c(1)$$

$$\Rightarrow \boxed{c = \frac{-E}{R}}$$

$$i = \frac{E}{R} - \frac{E}{R} e^{-\frac{R}{L}t}$$

$$\boxed{i = \frac{E}{R}\left(1 - e^{-R/L\,t}\right)}$$

Given that $i = \frac{1}{2}\frac{E}{R}$, $t = ?$

$$\frac{1}{2}\frac{\cancel{E}}{\cancel{R}} = \frac{\cancel{E}}{\cancel{R}}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$e^{-\frac{R}{L}t} = 1 - 1/2$$

$$e^{-R/L\,t} = 1/2$$

$$-\frac{R}{L}t = \log 1/2$$

$$t = -\frac{L}{R}(\log 1 - \log 2)$$

$$t = -\frac{L}{R}(0 - \log 2)$$

$$t = \frac{+L \log 2}{R} \text{ sec.}$$

(3) By using Kirchoff's Law the equ^n of the LR circuit is $L\frac{di}{dt} + Ri = E$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}e^{-at} \to (1)$$

Equ^n (1) is in linear form

$$P = \frac{R}{L}, \quad Q = \frac{E}{L}e^{-at}$$

$$\text{If } e^{\int p(t)dt} = e^{\int \frac{R}{L}dt}$$

$$= e^{\frac{R}{L}\int (1)dt}$$

$$= \underline{\underline{e^{\frac{R}{L}t}}}$$

Now the sol^n of equ^n (1) is

$$i \cdot e^{\frac{R}{L}t} = \int \frac{E}{L} e^{-at} e^{R/L\,t} + C$$

$$= \frac{E}{\cancel{L}} \frac{e^{R/L\,t}}{R/\cancel{L}} + C$$

$$= \frac{E}{R} e^{R/L\,t} + C$$

$$i \cdot \cancel{e^{R/L\,t}} = \cancel{e^{R/L\,t}}\left(\frac{E}{R} + C \cdot e^{-R/L\,t}\right)$$

$$\boxed{i = \frac{E}{R} + Ce^{-R/L\,t}}$$

Initially, $t = 0$, $i = 0$.

$$0 = \frac{E}{R} + C \cdot e^{-\frac{R}{L}(0)}$$

$$-\frac{E}{R} = C e^{(0)}$$

$$\Rightarrow \boxed{C = \frac{-E}{R}}$$

$$i = \frac{E}{R} + -\frac{E}{R} e^{-R/L\, t}$$

$$\boxed{i = \frac{E}{R}\left(1 - e^{-R/L\, t}\right)}$$

Given that

$$i \cdot e^{R/L\, t} = \int \frac{E}{L} e^{-at} e^{R/L\, t} dt + C$$

$$= \frac{E}{L} \int e^{R/L\, t - at} dt + C$$

$$= \frac{E}{L} \int e^{(R/L - a)t} dt + C$$

$$= \frac{E}{L} \frac{e^{(R/L - a)t}}{(R/L - a)} + C$$

$$= \frac{E}{\cancel{L}} \frac{e^{(R/L - a)t}}{\left(\frac{R - aL}{\cancel{L}}\right)} + C$$

$$= \frac{E}{R - aL} e^{(R/L - a)t} + C$$

$$= \frac{E}{R - aL} e^{R/L\, t} \cdot e^{-at} + C$$

$$i \cdot \cancel{e^{R/L\, t}} = \cancel{e^{R/L\, t}} \left[\frac{E}{R - aL} e^{-at} + C \cdot e^{-R/L\, t}\right]$$

$$i = \left(\frac{E}{R - aL} e^{-at} + C \cdot e^{-R/L\, t}\right)$$

Initially $t = 0$ and $i = 0$

$$0 = \frac{E}{R - aL} e^{-a(0)} + C \cdot e^{-R/L(0)}$$

$$-\frac{E}{R - aL} (e)^{(0)} = C \cdot e^{(0)}$$

$$\frac{-E}{R - aL} = C(1)$$

$$\boxed{C = \frac{-E}{R - aL}}$$

$$i = \frac{E}{R-aL} \cdot e^{-at} - \frac{E}{R-aL} e^{-R/L\, t}$$

$$i = \frac{E}{R-aL} \left[e^{-at} - e^{-R/L\, t} \right]$$

(4) By using Kirchoff's Law the equⁿ of the LR circuit is $L\frac{di}{dt} + Ri = E$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{E}{L}$$

Given that $L\frac{di}{dt} + Ri = 200 \cdot \cos(300t)$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{200 \cdot \cos(300t)}{L} \to (\emptyset)$$

Given that $R = 100$, $L = 0.05$

$$\frac{di}{dt} + \frac{100}{0.05} i = \frac{200 \cdot \cos(300t)}{0.05}$$

$$\frac{di}{dt} + 2000\, i = 4000 \cdot \cos(300t) \to (1)$$

Equⁿ (1) is in linear form.

$P = 2000$ and $Q = 4000 \cos(300t)$.

$$I.F\ e^{\int 2000 dt} = e^{2000 \int (1) dt} = e^{2000t}$$

$$i \cdot e^{2000t} = \int 4000 \cdot \cos(300t)\, e^{2000t}\, dt + c$$

$$= 4000 \int \cos(300t) \cdot e^{2000t}\, dt + c.$$

$$= 4000 \int e^{2000t} \cos(300t) + c.$$

$$= 4000 \cdot \left[\frac{e^{(2000)t}}{(2000)^2 + (300)^2} \left[2000 \cos(300)t + 300 \cdot \sin(300)t \right] + c \right]$$

$$i \cdot e^{2000t} = \cancel{4000} \left[\frac{e^{(2000)t}}{4090\cancel{000}} \left[2000 \cos(300)t + 300 \sin(300)t \right] + c \right]$$

$$= \frac{4}{4090} e^{(2000)t} \left[2000 \cdot \cos(300)t + 300 \sin(300)t \right] + c$$

$$= e^{(2000)t} \left[\frac{4 \times 200\cancel{0}}{409\cancel{0}} \cos(300)t + \frac{4 \times 30\cancel{0}}{409\cancel{0}} \sin(300)t \right] + c$$

$$= e^{(2000)t} \left[\frac{40 \times 20}{409} \cos(300t) + \frac{40 \times 3}{409} \sin(300)t \right] + c$$

$$i \cdot e^{(2000)t} = e^{(2000)t} \frac{40}{409}\left[\left[20 \cos(300)t + 3 \cdot \sin(300)t\right] + C e^{(2000)}\right.$$

$$i = \frac{40}{409}\left[20 \cos(300)t + 3 \sin(300)t\right] + C \cdot e^{-(2000)t}$$

Given that $i = 0$ and $t = 0$.

$$0 = \frac{40}{409}\left[20 \cdot \cos(300)(0) + 3 \cdot \sin(300)(0)\right] + C \cdot e^{-(2000)(0)}$$

$$0 = \frac{40}{409}\left(20 \cdot \cos(0) + 3 \cdot \sin(0)\right) + C \cdot e^{(0)}$$

$$0 = \frac{40}{409}\left(20(1) + 3(0)\right) + C(1)$$

$$0 = \frac{40}{409}(20 + 0) + C$$

$$\boxed{C = \frac{-40 \times 20}{409}}$$

$$i = \frac{40}{409}\left[20 \cos(300)t + 3 \cdot \sin(300)t\right] - \frac{40 \times 20}{409} e^{-(2000)t}$$

$$i = \frac{40}{409}\left[20 \cos(300)t + 3 \cdot \sin(300)t - 20 \cdot e^{-(2000)t}\right]$$

$$i = \frac{40}{409}\left[20\left(\cos(300)t - e^{-(2000)t}\right) + 3 \cdot \sin(300)t\right]$$

(2) By using Kirchoff's Law the equn of the LR circuit is $L\frac{di}{dt} + Ri = E$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{E}{L}$$

Given that $R = 100$, $L = 0.5$, E 20

$$\frac{di}{dt} + \frac{100}{0.5} i = \frac{20}{0.5}$$

$$\frac{di}{dt} + 200 \cdot i = 40 \quad \rightarrow ①$$

Equn ① is in linear form.

$P = 200$ and $Q = 40$

$$I.F \; e^{\int 200\, dt} = e^{200 \int (1) dt}$$

$$= e^{200t}$$

$$i \cdot e^{200t} = \int 40 \cdot e^{200t}\, dt + c$$

$$= 40 \int e^{200t}\, dt + c$$

$$= \cancel{40} \frac{e^{200t}}{\cancel{200}_{5}} + c$$

$$i \cdot e^{200t} = \frac{1}{5} \cdot e^{200t} + c$$

$$i \cdot \cancel{e^{200t}} = \cancel{e^{200t}} \left(\frac{1}{5} + c \cdot e^{-200t}\right)$$

$$i = \frac{1}{5} + c \cdot e^{-200t}.$$

Initially $t=0$ and $i=0$

$$0 = \frac{1}{5} + c \cdot e^{-200(0)}$$

$$-\frac{1}{5} = c \cdot e^{(0)}$$

$$-1/5 = c(1)$$

$$\Rightarrow \boxed{c = -1/5}$$

$$i = \frac{1}{5} - \frac{1}{5} e^{-200t}$$

$$i = \frac{1}{5}\left(1 - e^{-200t}\right).$$

Law of Growth:

(3) We have $y = ce^{Kt} \rightarrow (1)$

Initially $t=0$ and $y = N$

$$N = ce^{K(0)}$$

$$N = c\,e^{(0)}$$

$$= c(1)$$

$$\Rightarrow \boxed{c = N}$$

$$y = N \cdot e^{Kt} \rightarrow (2)$$

and $t = 2$ and $y = 3N$.

$$3\cancel{N} = \cancel{N} e^{K(2)}$$

$$2K = \log 3$$

$$K = \frac{1}{2} \log 3$$

$$K = 0.549306144$$

$$\boxed{K = 0.549}$$

$$y = N \cdot e^{(0.549)t} \rightarrow (3)$$

And also $t = ?$ and $y = 100N$

$$100\cancel{N} = \cancel{N} e^{(0.549)t}$$

$$e^{(0.549)t} = 100$$

$$(0.549)\, t = \log 100$$

$$t = \frac{1}{0.549} \log(100)$$

$$t = 8.388288135$$

$$\boxed{t \cong 8}$$